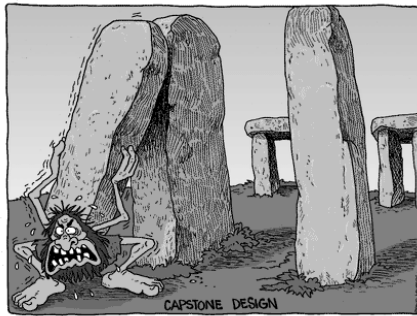


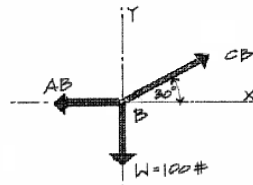
lecture
four



rigid body equilibrium

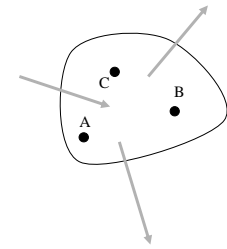
Free Body Diagram

- FBD (sketch)
- tool to see all forces on a body or a point including
 - external forces
 - weights
 - force reactions
 - external moments
 - moment reactions
 - internal forces



Equilibrium

- rigid body
 - doesn't deform
 - coplanar force systems



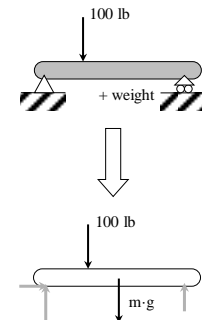
• static: $R_x = \sum F_x = 0$

$$R_y = \sum F_y = 0$$

$$M = \sum M = 0$$

Free Body Diagram

- determine body
- FREE it from:
 - ground
 - supports & connections
- draw all external forces acting ON the body
 - reactions
 - applied forces
 - gravity

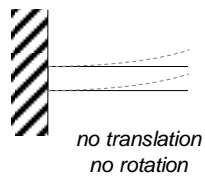
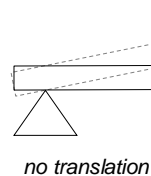
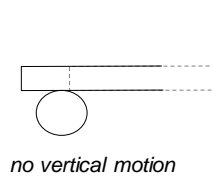


Free Body Diagram

- sketch FBD with relevant geometry
- resolve each force into components
 - known & unknown angles – name them
 - known & unknown forces – name them
 - known & unknown moments – name them
- are any forces related to other forces?
- for the unknowns
- write only as many equilibrium equations as needed
- solve up to 3 equations

Reactions on Rigid Bodies

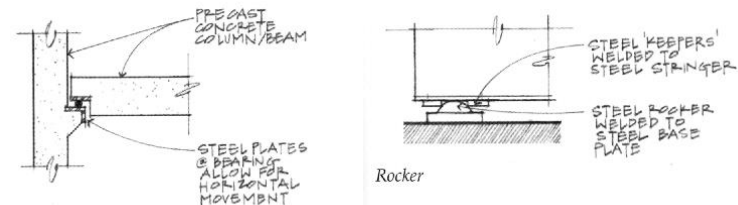
- result of applying force
- unknown size
- connection or support type
 - known direction
 - related to motion prevented



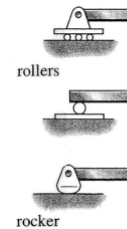
Free Body Diagram

- solve equations
 - most times 1 unknown easily solved
 - plug into other equation(s)
- common to have unknowns of
 - force magnitudes
 - force angles
 - moment magnitudes

Supports and Connections



Roller

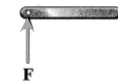


rollers

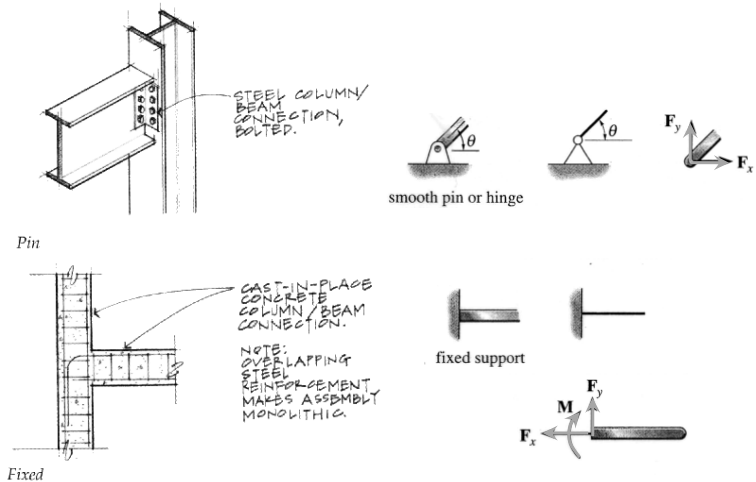
rocker



Rocker



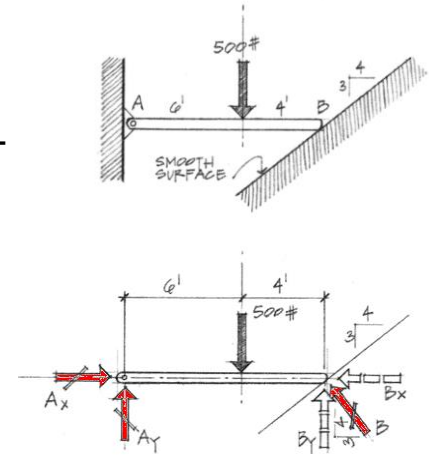
Supports and Connections



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FBD Example

- 500 lb known
- pin – A_x, A_y
- smooth surface – B at 4:3
- 3 equations
- sum moments at – A? – B? (B_x)



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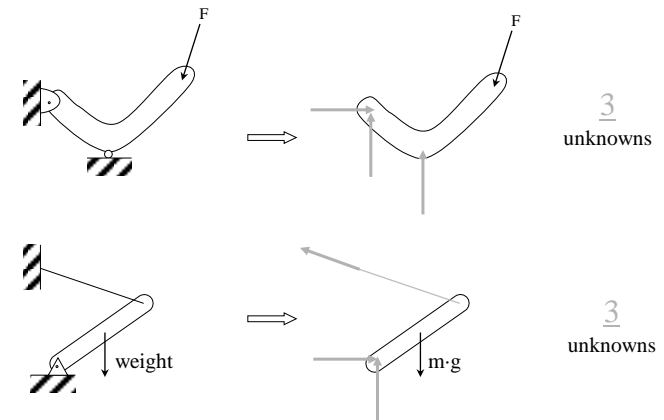
Moment Equations

- sum moments at intersection where the most forces intersect
- multiple moment equations may not be useful
- combos:

$$\begin{matrix} \sum F_x = 0 & \sum F = 0 & \sum M_1 = 0 \\ \sum F_y = 0 & \sum M_1 = 0 & \sum M_2 = 0 \\ \sum M_1 = 0 & \sum M_2 = 0 & \sum M_3 = 0 \end{matrix}$$

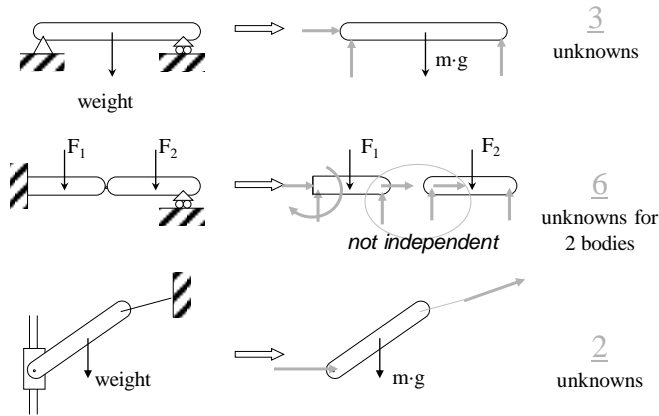
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Recognizing Reactions



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Recognizing Reactions



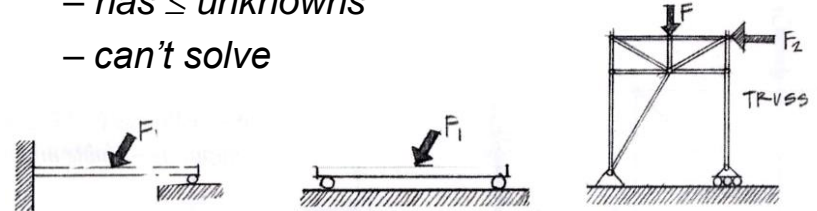
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Constraints

- **completely constrained**
 - doesn't move
 - may not be statically determinate
- **improperly or partially constrained**
 - has \leq unknowns
 - can't solve



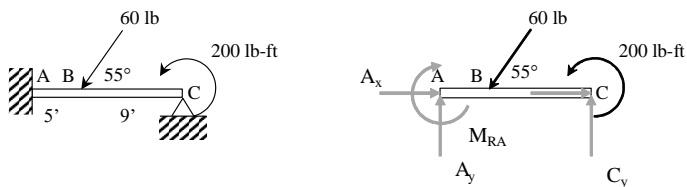
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Constraints

- **overconstrained**
 - won't move
 - can't be solved with statics
 - statically indeterminate to n^{th} degree

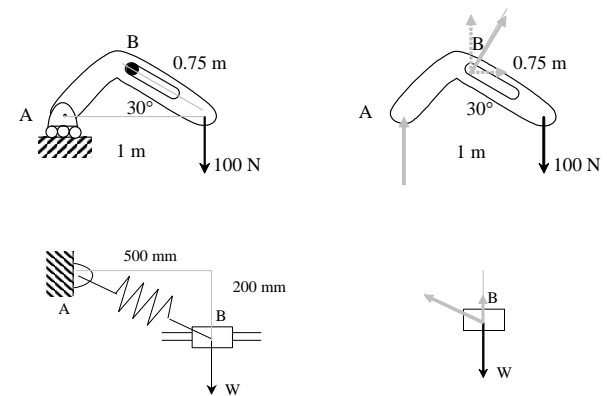


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Partial Constraints



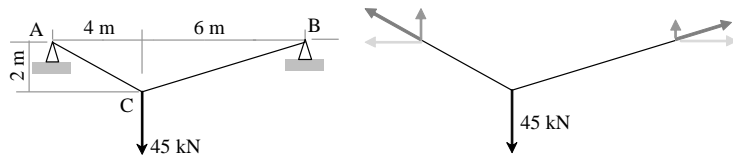
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Cable Reactions

- **equilibrium:**
 - more reactions (4) than equations
 - but, we have slope relationships
 - x component the same everywhere



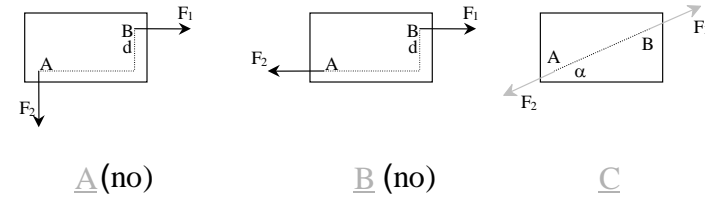
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Two Force Rigid Bodies

- **equilibrium:**
 - forces in line, equal and opposite



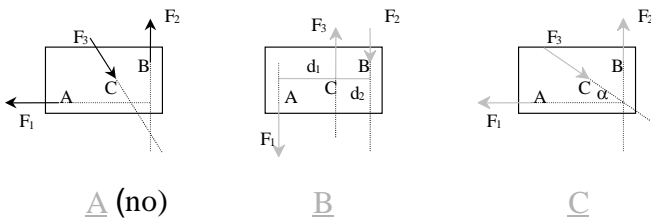
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Three Force Rigid Bodies

- **equilibrium:**
 - concurrent or parallel forces
 - beams!*

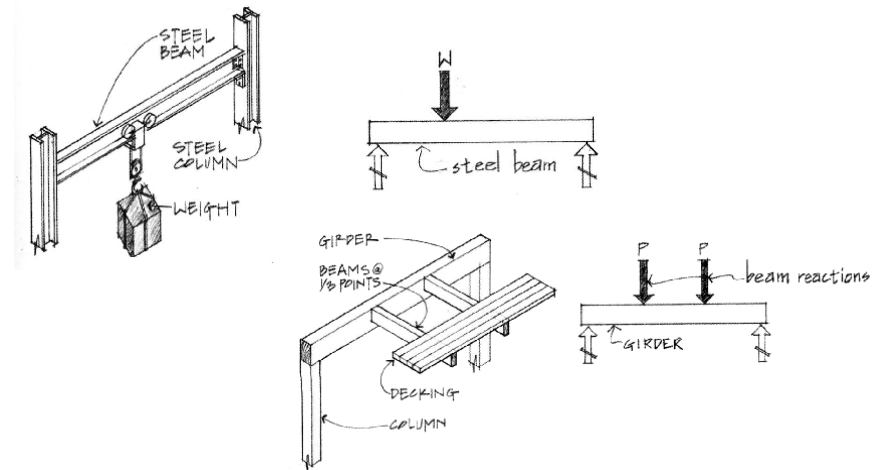


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Concentrated Loads

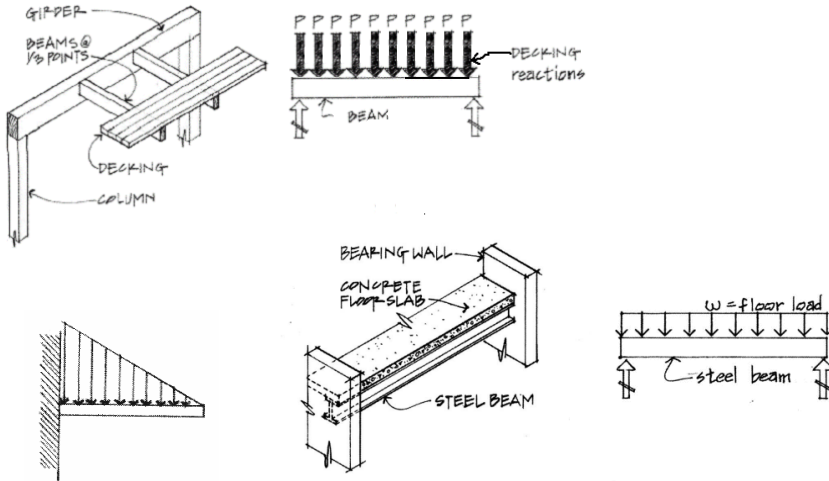


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Distributed Loads



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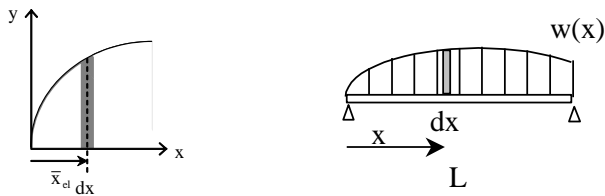
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Equivalent Force Systems

- replace forces by resultant
- place resultant where $M = 0$
- using calculus and area centroids

$$W = \int_0^L w dx = \int dA_{\text{loading}} = A_{\text{loading}}$$



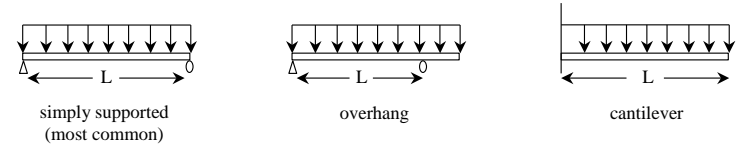
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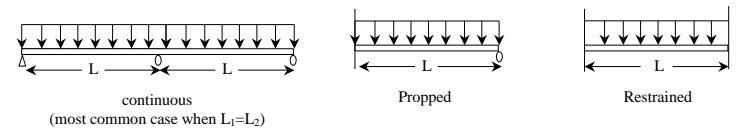
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Beam Supports

- *statically determinate*



- *statically indeterminate*



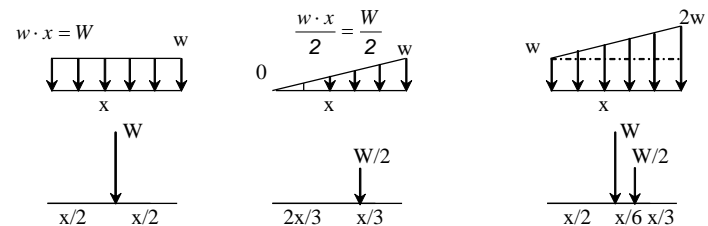
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Load Areas

- area is width x "height" of load
- w is load per unit length
- W is total load



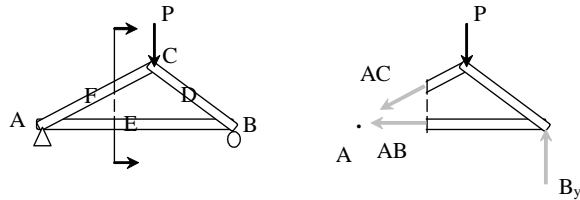
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Method of Sections

- relies on internal forces being in equilibrium on a section
- cut to expose 3 or less members
- coplanar forces $\rightarrow \sum M = 0$ too



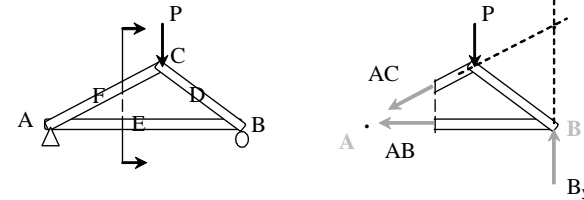
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Method of Sections

- joints on or off the section are good to sum moments
- quick for few members
- not always obvious where to cut or sum



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