

## Examples: Reinforced Concrete

### Example 1

A simply supported beam 20 ft long carries a service dead load of 300 lb/ft and a live load of 500 lb/ft. Design an appropriate beam (for flexure only). Use grade 40 steel and concrete strength of 5000 psi.

SOLUTION:

Find the design moment,  $M_u$ , from the factored load combination of 1.2D + 1.6L. It is good practice to guess a beam size to include self weight in the dead load, because "service" means dead load of everything except the beam itself.

Guess a size of 10 in x 12 in. Self weight for normal weight concrete is the density of 150 lb/ft<sup>3</sup> multiplied by the cross section area:

$$\text{self weight} = 150 \frac{\text{lb}}{\text{ft}^3} (10\text{in})(12\text{in}) \cdot \left(\frac{1\text{ft}}{12\text{in}}\right)^2 = 125 \text{ lb/ft}$$

$$w_u = 1.2(300 \text{ lb/ft} + 125 \text{ lb/ft}) + 1.6(500 \text{ lb/ft}) = 1310 \text{ lb/ft}^2$$

The maximum moment for a simply supported beam is  $\frac{wl^2}{8}$ :

$$M_u = \frac{w_u l^2}{8} = \frac{1310 \frac{\text{lb}}{\text{ft}} (20\text{ft})^2}{8} = 65,500 \text{ lb-ft}$$

$$M_n \text{ required} = M_u / \phi = \frac{65,500 \text{ lb-ft}}{0.9} = 72,778 \text{ lb-ft}$$

To use the design chart aid, find  $R_n = \frac{M_n}{bd^2}$ ,

estimating that  $d$  is about 1.75 inches less than  $h$ :

$$d = 12\text{in} - 1.75\text{in} = 10.25\text{in} \quad (\text{no stirrups assuming a \#4 bar})$$

$$R_n = \frac{72,778 \text{ lb-ft}}{(10\text{in})(10.25\text{in})^2} \cdot \left(12 \frac{\text{in}}{\text{ft}}\right) = 831 \text{ psi}$$

Maximum Reinforcement Ratio  $\rho$  for Singly Reinforced Rectangular Beams  
(tensile strain = 0.005) for which  $\phi$  is permitted to be 0.9

$f_y$	$f'_c = 3000 \text{ psi}$	$f'_c = 3500 \text{ psi}$	$f'_c = 4000 \text{ psi}$	$f'_c = 5000 \text{ psi}$	$f'_c = 6000 \text{ psi}$
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.80$	$\beta_1 = 0.75$
40,000 psi	0.0203	0.0237	0.0271	0.0319	0.0359
50,000 psi	0.0163	0.0190	0.0217	0.0255	0.0287
60,000 psi	0.0135	0.0158	0.0181	0.0213	0.0239
$f_y$	$f'_c = 20 \text{ MPa}$	$f'_c = 25 \text{ MPa}$	$f'_c = 30 \text{ MPa}$	$f'_c = 35 \text{ MPa}$	$f'_c = 40 \text{ MPa}$
	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.85$	$\beta_1 = 0.81$	$\beta_1 = 0.77$
300 MPa	0.0181	0.0226	0.0271	0.0301	0.0327
350 MPa	0.0155	0.0194	0.0232	0.0258	0.0281
400 MPa	0.0135	0.0169	0.0203	0.0226	0.0245
500 MPa	0.0108	0.0135	0.0163	0.0181	0.0196

**Table 3.7.1**  
Total Areas for Various Numbers of Reinforcing Bars

Bar Size	Nominal		Number of Bars									
	Diameter (in.)	Weight (lb/ft)	1	2	3	4	5	6	7	8	9	10
#3	0.375	0.376	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
#4	0.500	0.668	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
#5	0.625	1.043	0.31	0.62	0.93	1.24	1.55	1.86	2.17	2.48	2.79	3.10
#6	0.750	1.502	0.44	0.88	1.32	1.76	2.20	2.64	3.08	3.52	3.96	4.40
#7	0.875	2.044	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
#8	1.000	2.670	0.79	1.58	2.37	3.16	3.95	4.74	5.53	6.32	7.11	7.90
#9	1.128	3.400	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
#10	1.270	4.303	1.27	2.54	3.81	5.08	6.35	7.62	8.89	10.16	11.43	12.70
#11	1.410	5.313	1.56	3.12	4.68	6.24	7.80	9.36	10.92	12.48	14.04	15.60
#14*	1.693	7.65	2.25	4.50	6.75	9.00	11.25	13.50	15.75	18.00	20.25	22.50
#18*	2.257	13.60	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00

\* #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

**Example 1 (continued)**

$\rho$  corresponds to approximately 0.023 (which is less than that for 0.005 strain of 0.0319), so the estimated area required,  $A_s$ , can be found:

$$A_s = \rho b d = (0.023)(10\text{in})(10.25\text{in}) = 2.36 \text{ in}^2$$

The number of bars for this area can be found from handy charts.

(Whether the number of bars actually fit for the width with cover and space between bars must also be considered. If you are at  $\rho_{max}$  do not choose an area bigger than the maximum!)

Try  $A_s = 2.37 \text{ in}^2$  from 3#8 bars

$$d = 12 \text{ in} - 1.5 \text{ in (cover)} - \frac{1}{2} (8/8 \text{ in diameter bar}) = 10 \text{ in}$$

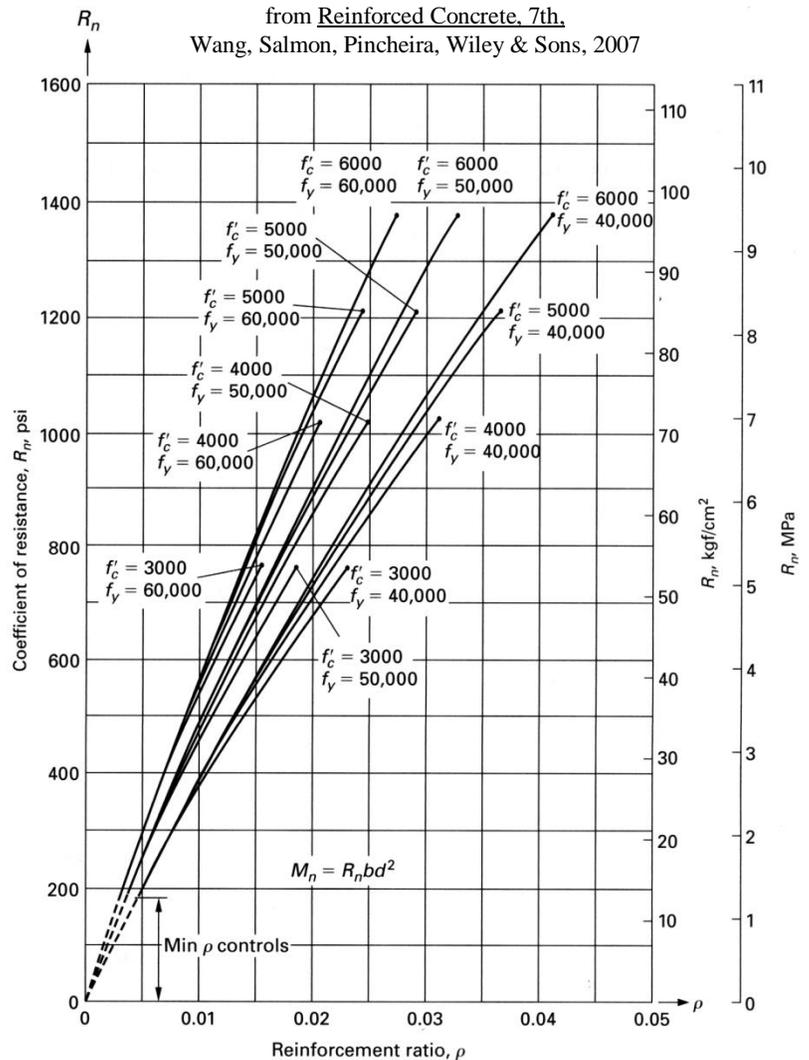
Check  $\rho = 2.37 \text{ in}^2 / (10 \text{ in})(10 \text{ in}) = 0.0237$  which is less than  $\rho_{max-0.005} = 0.0319$  OK (We can still use 0.9 for  $\phi$ . Never exceed  $\rho_{max-0.004}$ ! That is an over reinforced beam!!)

Find the moment capacity of the beam as designed,  $\phi M_n$

$$a = A_s f_y / 0.85 f'_c b = 2.37 \text{ in}^2 (40 \text{ ksi}) / [0.85(5 \text{ ksi})10 \text{ in}] = 2.23 \text{ in}$$

$$\phi M_n = \phi A_s f_y (d - a/2) = 0.9(2.37 \text{ in}^2)(40 \text{ ksi}) \left( 10 \text{ in} - \frac{2.23 \text{ in}}{2} \right) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 63.2 \text{ k-ft} \not> 65.5 \text{ k-ft needed (not OK)}$$

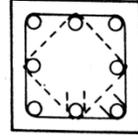
So, we can increase  $h$  to 13 in, and  $\phi M_n = 70.3 \text{ k-ft}$  (OK). Or increase  $A_s$  to 2 # 10's (2.54 in<sup>2</sup>), for a = 2.39 in and  $\phi M_n$  of 67.1 k-ft (OK). Don't exceed  $\rho_{max-0.005}$  if you want to use  $\phi=0.9$



**Figure 3.8.1** Strength curves ( $R_n$  vs  $\rho$ ) for singly reinforced rectangular sections. Upper limit of curves is at  $\rho_{max}$  (tensile strain of 0.004)

Example 2

Determine the capacity of a 16" x 16" column with 8- #10 bars, tied. Grade 40 steel and 4000 psi concrete.



SOLUTION:

Find  $\phi P_n$ , with  $\phi=0.65$  and  $P_n = 0.80P_o$  for tied columns and

$$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$$

Steel area (found from reinforcing bar table for the bar size):

$$A_{st} = 8 \text{ bars} \times (1.27 \text{ in}^2) = 10.16 \text{ in}^2$$

Concrete area (gross):

$$A_g = 16 \text{ in} \times 16 \text{ in} = 256 \text{ in}^2$$

**ASTM STANDARD REINFORCING BARS**

Bar size, no.	Nominal diameter, in.	Nominal area, in. <sup>2</sup>	Nominal weight, lb/ft
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Grade 40 reinforcement has  $f_y = 40,000$  psi and  $f'_c = 4000$ psi

$$\phi P_n = (0.65)(0.80)[0.85(4000 \text{ psi})(256 \text{ in}^2 - 10.16 \text{ in}^2) + (40,000 \text{ psi})(10.16 \text{ in}^2)] = 646,026 \text{ lb} = 646 \text{ kips}$$

**Example 3**

Design a 10 ft long circular spiral column for a braced system to support the service dead and live loads of 300 k and 460 k, respectively, and the service dead and live moments of 100 ft-k each. The moment at one end is zero. Use  $f'_c = 4,000$  psi and  $f_y = 60,000$  psi.

**Solution**

$$1. P_u = 1.2(300) + 1.6(460) = 1096 \text{ k}$$

$$M_u = 1.2(100) + 1.6(100) = 280 \text{ ft-k}$$

2. Assume  $\rho_g = 0.01$ , from Equation 16.10:

$$A_g = \frac{P_u}{0.60[0.85f'_c(1 - \rho_g) + f_y\rho_g]}$$

$$= \frac{1096}{0.60[0.85(4)(1 - 0.01) + 60(0.01)]}$$

$$= 460.58 \text{ in.}^2$$

$$\frac{\pi h^2}{4} = 460.58$$

or  $h = 24.22$  in.  
Use  $h = 24$  in.,  $A_g = 452$  in.<sup>2</sup>

3. Assume #9 size of bar and 3/8 in. spiral center-to-center distance  
 $= 24 - 2(\text{cover}) - 2(\text{spiral diameter}) - 1$  (bar diameter)  
 $= 24 - 2(1.5) - 2(3/8) - 1.128 = 19.12$  in.

$$\gamma = \frac{19.12}{24} = 0.8$$

ACI 20.6: Concrete exposed to earth or weather:  
 No. 6 through No. 18 bars..... 2 in. minimum

Use the interaction diagram Appendix D.21

$$4. K_n = \frac{P_u}{\phi f'_c A_g} = \frac{1096}{(0.75)(4)(452)} = 0.808$$

$$R_n = \frac{M_u}{\phi f'_c A_g h} = \frac{3360}{(0.75)(4)(452)(24)} = 0.103$$

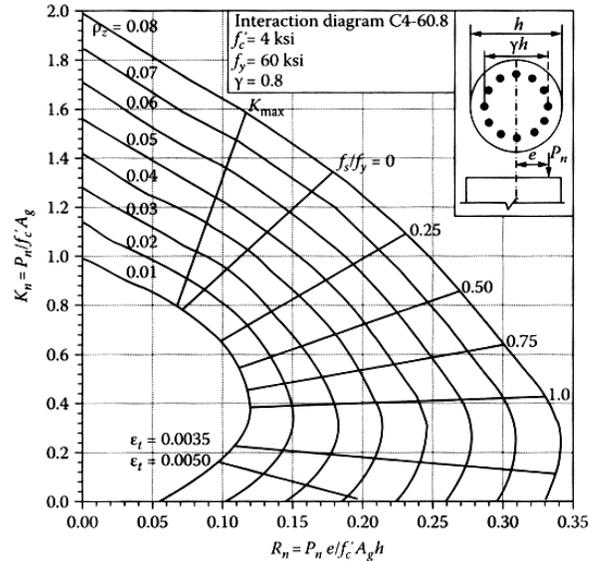
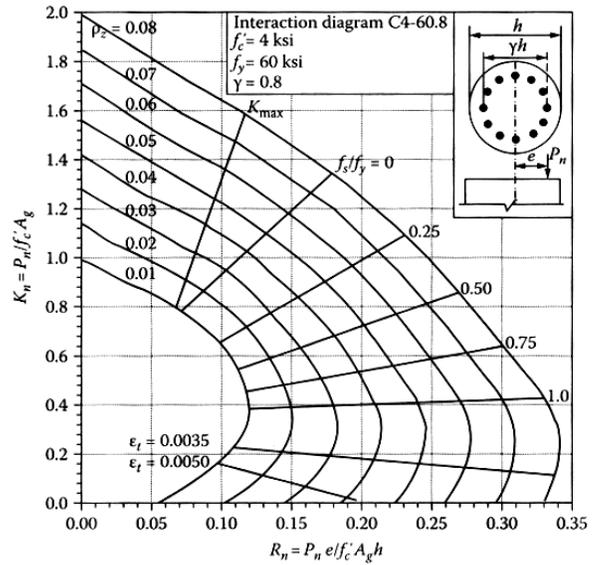


Figure D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

Example 3 (continued)

5. At the intersection point of  $K_n$  and  $R_n$ ,  $\rho_g = 0.02$

6. The point is above the strain line = 1, hence  $\phi = 0.7$  **OK**



**Figure D.21** Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

7.  $A_{st} = (0.02)(452) = 9.04 \text{ in.}^2$

From Appendix D.2, select 12 bars of #8,  $A_{st} = 9.48 \text{ in.}^2$

From Appendix D.14 for a core diameter of  $24 - 3 = 21 \text{ in.}$ , 17 bars of #8 can be arranged in a row

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Total Areas for Various Numbers of Reinforcing Bars

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			1	2	3	4	5	6	7	8	9	10
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\* #14 and #18 bars are used primarily as column reinforcement and are rarely used in beams.

9.  $K/l = 1, l = 10 \times 12 = 120 \text{ in.}, r = 0.25(24) = 6 \text{ in.}$

$$\frac{Kl}{r} = \frac{1(120)}{6} = 20$$

$$\left( \frac{M_1}{M_2} \right) = 0$$

$$34 - 12 \left( \frac{M_1}{M_2} \right) = 34$$

since  $(Kl/r) < 34$ , short column.

ACI 6.2: In nonsway frames it shall be permitted to ignore slenderness effects for compression members that satisfy: