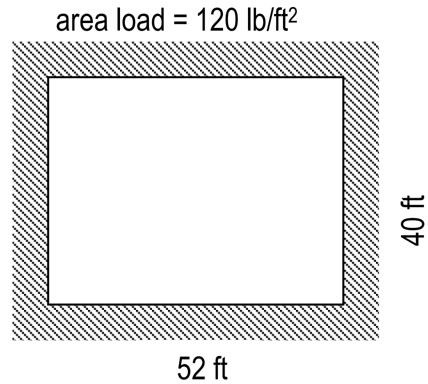
Examples:
Plate and Grids

Example 1

What is the maximum positive and negative bending moments developed in a 52 x 40 ft fully fixed plate that carries a load of 120 lb/ft^2?



SOLUTION:

The aspect ratio of the side lengths, *a/b*, must be determined and an appropriate coefficient chart must be found:

a/b = 52/40 = 1.3 (no units, and *a* is always the *bigger* number).

BENDING MOMENTS IN RECTANGULAR PLATES

Aspect ratio $\frac{a}{b}$	Simply supported on all four sides	Fixed on all four sides	Corner slabs fixed on two adjacent sides and free on two sides
1.0	$C_a = +0.0479$ $C_b = +0.0479$	$C_a = +0.0231$, $C_a = -0.0513$ $C_b = +0.0231$, $C_b = -0.0513$	$C_a = -0.29$ $C_b = -0.29$
1.3	$C_a = +0.0298$ $C_b = +0.0694$	$C_a = +0.0131$, $C_a = -0.0333$ $C_b = +0.0327$, $C_b = -0.0687$	$C_a = -0.35$ $C_b = -0.35$
1.5	$C_a = +0.0221$ $C_b = +0.0812$	$C_a = +0.0090$, $C_a = -0.0253$ $C_b = +0.0368$, $C_b = -0.0757$	$C_a = -0.37$ $C_b = -0.37$
2.0	$C_a = +0.0116$ $C_b = +0.1017$	$C_a = +0.0039$, $C_a = -0.0143$ $C_b = +0.0412$, $C_b = -0.0829$	$C_a = -0.43$ $C_b = -0.43$

Note: In all cases,

$$M_a = C_a w a^2$$

$$M_b = C_b w b^2$$

The coefficients for moment for the *a* side length and *b* side length for fixed support all sides and a/b = 1.3 are:

C_a = +0.0131 and C_a = -0.0333 C_b = +0.0327 and C_b = -0.0687

The maximum moments are calculated with the formula in the table:

$$M_a(positive) = C_a wa^2 = 0.0131(120\ {}^{lb}\!/_{ft^2})(52\ ft)^2 = 4251\ \tfrac{lb-ft}{ft}$$

$$M_a(negative) = C_a wa^2 = -0.0333(120\ {}^{lb}\!/_{ft^2})(52\ ft)^2 = -10{,}805\ \tfrac{lb-ft}{ft}$$

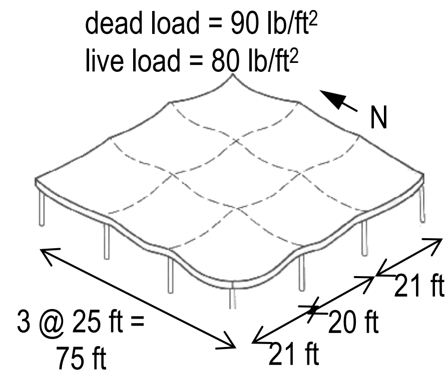
$$M_b(positive) = C_b wb^2 = 0.0327(120\ {}^{lb}\!/_{ft^2})(40\ ft)^2 = 6278\ \tfrac{lb-ft}{ft}$$

$$M_b(negative) = C_b wb^2 = -0.0687(120\ {}^{lb}\!/_{ft^2})(40\ ft)^2 = -13{,}190\ \tfrac{lb-ft}{ft}$$

Example 2

A two-way interior-bay flat plate (concrete) with the dimensions shown supports a live loading of 80 lb/ft^2 and has a dead load of 90 lb/ft^2. The columns can be assumed to be 18 inches square. Determine the design moments based on ACI-318, (ASCE-7) and the Direct Design method.

Also compare design moments for an exterior-interior bay

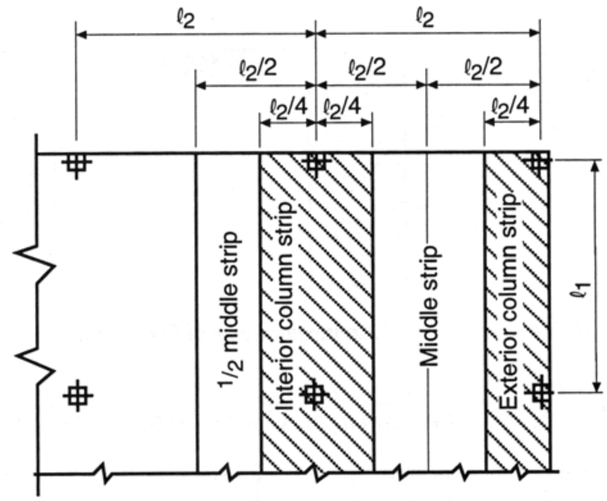


SOLUTION:
Determine the distributed load combinations:

$w_u = 1.2D + 1.6L$ = 1.2(90 lb/ft^2) + 1.6(80 lb/ft^2) = 236 lb/ft^2

Determine the clear span length for the N-S direction:

$\ell_n = \ell_1$ – ½ column width – ½ column width
= 25 ft – ½ (18 in/12 in/ft) – ½ (18 in/12 in/ft) = 23.5 ft



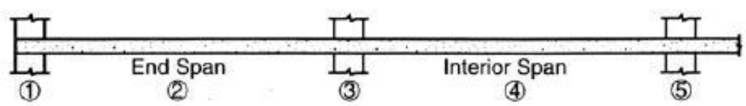
(a) Column strip for $\ell_2 \leq \ell_1$

Because ℓ_2 is not the same width on either side of an interior panel, it is taken as the average = (21 ft + 20 ft)/2 = 20.5 ft.

Total moment (to distribute to middle and interior column strip):

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236\, ^{lb}/_{ft^2})(20.5\,ft)(23.5\,ft)^2}{8} = 333{,}973^{lb-ft}$$

Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns



Slab Moments	End Span 1 Exterior Negative	End Span 2 Positive	End Span 3 First Interior Negative	Interior Span 4 Positive	Interior Span 5 Interior Negative
Total Moment	0.26 M$_o$	0.52 M$_o$	0.70 M$_o$	0.35 M$_o$	0.65 M$_o$
Column Strip	0.26 M$_o$	0.31 M$_o$	0.53 M$_o$	0.21 M$_o$	0.49 M$_o$
Middle Strip	0	0.21 M$_o$	0.17 M$_o$	0.14 M$_o$	0.16 M$_o$

Note: All negative moments are at face of support.

<u>Interior Column Strip</u> ($\ell_2 \leq \ell_1$):

The column strip width is ¼ the smaller of ℓ_2 either side of the column:

strip width = ¼ (21 ft) + ¼ (20 ft) = 10.25 ft

Example 2 (*continued*)

From Table 4.2, the maximum positive moment occurs in an end span:

$M(positive) = 0.31M_o = (0.31)(333{,}973^{lb-ft}) = 103{,}532^{lb-ft}$, distributed over 10.25 ft = 103,532 lb-ft/(10.25 ft) = 10,101 lb-ft/ft

The positive design moment for an interior span is:

$M(positive) = 0.21M_o = (0.21)(333{,}973^{lb-ft}) = 70{,}134^{lb-ft}$, distributed over 10.25 ft = 70,134 lb-ft/(10.25 ft) = = 6842 lb-ft/ft

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$M(negative) = 0.53M_o = (0.53)(333{,}973^{lb-ft}) = 177{,}006^{lb-ft}$, distributed over 10.25 ft =177,006 lb-ft/(10.25 ft) = = 17,269 lb-ft/ft

The negative design moment at the exterior of an end span is:

$M(negative) = 0.26M_o = (0.26)(333{,}973^{lb-ft}) = 86{,}833^{lb-ft}$, distributed over 10.25 ft = 86,833 lb-ft/(10.25 ft) = = 8472 lb-ft/ft

The negative design moment for an interior span is:

$M(negative) = 0.49M_o = (0.49)(333{,}973^{lb-ft}) = 163{,}647^{lb-ft}$, distributed over 10.25 ft = 163,647 lb-ft/(10.25 ft) = = 15,966 lb-ft/ft

<u>Middle Strip:</u>

The width is the remaining width of ℓ_2 between column strips:

strip width = 21 ft – ¼ (20 ft) – ¼ (21 ft) = 10.75 ft

From Table 4.2, the maximum positive moment occurs in an end span:

$M(positive) = 0.21M_o = (0.21)(333{,}973^{lb-ft}) = 70{,}134^{lb-ft}$, distributed over 10.75 ft = 70,134 lb-ft/(10.75 ft) = = 6524 lb-ft/ft

The positive design moment for an interior span is:

$M(positive) = 0.14M_o = (0.14)(333{,}973^{lb-ft}) = 46{,}756^{lb-ft}$, distributed over 10.75 ft = 46,756 lb-ft/(10.75 ft) = = 4349 lb-ft/ft

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$M(negative) = 0.17M_o = (0.17)(333{,}973^{lb-ft}) = 56{,}775^{lb-ft}$, distributed over 10.75 ft = 56,775 lb-ft/(10.75 ft) = = 5281 lb-ft/ft

Example 2 (*continued*)

There is no negative design moment at the exterior of an end span.

The negative design moment for an interior span is:

$M(negative) = 0.16M_o = (0.16)(333{,}973^{lb-ft}) = 53{,}436^{lb-ft}$, distributed over 10.75 ft = 53,436 lb-ft/(10.75 ft) = = 4971 lb-ft/ft

Exterior Column Strip:

The value to use for ℓ_2 for an edge strip includes the distance to the outside of the columns = 21 ft+ ½ (18 in/12 in/ft) = 21.75 ft

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236\ {}^{lb}/_{ft^2})(21.75\ ft)(23.5\ ft)^2}{8} = 354{,}337^{\ lb-ft}$$

The width is ¼ l_2 one side of the column plus the distance to the slab edge:

strip width = ¼ (21 ft) + ½ (18 in/12 in/ft) = 6 ft

So a comparison to the interior column strip maximum positive moment occurring in an end span is:

$M(positive) = 0.31M_o = (0.31)(354{,}337^{lb-ft}) = 109{,}844^{lb-ft}$, distributed over 6 ft = 109,844 lb-ft/(6 ft) = 18,307 lb-ft/ft (as opposed to 10,101 lb-ft/ft)

For the E-W direction:

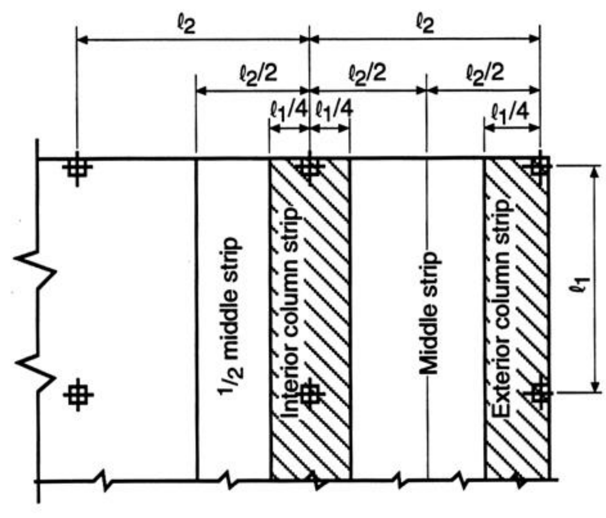
Because the adjacent spans are not the same length, the longer span, which is the END span will be larger:

ℓ_n = ℓ_1 – ½ column width – ½ column width
= 21 ft – ½ (18 in/12 in/ft) – ½ (18 in/12 in/ft) =19.5 ft

Because ℓ_2 is 25 ft.

Total moment (to distribute to middle and interior column strip):

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236\ {}^{lb}/_{ft^2})(25\ ft)(19.5\ ft)^2}{8} = 280{,}434^{\ lb-ft}$$



(b) Column strip for $\ell_2 > \ell_1$

Interior Column Strip END Spans (ℓ_2 > ℓ_1):

The column strip width is ¼ the **smaller** of ℓ_1 and ℓ_2 either side of the column:

strip width = ¼ (21 ft) + ¼ (21 ft) = 10.5 ft

From Table 4.2, the maximum positive moment occurs in an end span:

$M(positive) = 0.31M_o = (0.31)(280{,}434^{lb-ft}) = 86{,}935^{lb-ft}$, distributed over 10.5 ft = 86,935 lb-ft/(10.5 ft) = 8279 lb-ft/ft

Example 2 (*continued*)

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$M(negative) = 0.53M_o = (0.53)(280{,}434^{lb-ft}) = 148{,}630^{lb-ft}$, distributed over 10.5 ft = 148,630 lb-ft/(10.5 ft) = = 14,155 lb-ft/ft

The negative design moment at the exterior of an end span is:

$M(negative) = 0.26M_o = (0.26)(280{,}434^{lb-ft}) = 72{,}913^{lb-ft}$, distributed over 10.5 ft = 72,913 lb-ft/(10.5 ft) = = 6944 lb-ft/ft

Middle Strip END Spans:

The width is the remaining width of l_2 between column strips:

strip width = 25 ft – ¼ (21 ft) – ¼ (21 ft) = 14.5 ft

From Table 4.2, the maximum positive moment occurs in an end span:

$M(positive) = 0.21M_o = (0.21)(280{,}434^{lb-ft}) = 58{,}891^{lb-ft}$, distributed over 14.5 ft = 58,891 lb-ft/(14.5 ft) = = 4061 lb-ft/ft

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

$M(negative) = 0.17M_o = (0.17)(280{,}434^{lb-ft}) = 47{,}674^{lb-ft}$, distributed over 14.5 ft = 47,674 lb-ft/(14.5 ft) = = 3288 lb-ft/ft

There is no negative design moment at the exterior of an end span.

Exterior Column Strip END Spans:

The value to use for l_2 for an edge strip includes the distance to the outside of the columns = 25 ft+ ½ (18 in/12 in/ft) = 25.75 ft

$$M_o = \frac{w_u \ell_2 \ell_n^2}{8} = \frac{(236\ ^{lb}/_{ft^2})(25.75\ ft)(19.5\ ft)^2}{8} = 288{,}847^{lb-ft}$$

The width is ¼ l_1 (because it is smaller than l_2) one side of the column plus the distance to the slab edge:

strip width = ¼ (21 ft) + ½ (18 in/12 in/ft) = 6 ft

So a comparison to the interior column END strip maximum positive moment occurring in an end span is:

$M(positive) = 0.31M_o = (0.31)(288{,}847^{lb-ft}) = 89{,}543^{lb-ft}$, distributed over 6 ft = 89,543 lb-ft/(6 ft) = 14,923 lb-ft/ft (as opposed to 8279 lb-ft/ft)

Example 2 (*continued*)

TABLE OF DESIGN MOMENTS

slab moments / ft	End Span			Interior Span	
	Exterior Negative	Positive	First Interior Negative	Positive	Interior Negative
NS column strip - interior	8472 lb-ft/ft	10,101 lb-ft/ft	17,269 lb-ft/ft	6842 lb-ft/ft	15,966 lb-ft/ft
NS middle strip	0	6524 lb-ft/ft	5281 lb-ft/ft	4349 lb-ft/ft	4971 lb-ft/ft
NS column strip - edge	15,355 lb-ft/ft	18,307 lb-ft/ft	31,300 lb-ft/ft	12,402 lb-ft/ft	28,937 lb-ft/ft
EW column strip –interior	6944 lb-ft/ft	8279 lb-ft/ft	14,155 lb-ft/ft	5048 lb-ft/ft	11,779 lb-ft/ft
EW middle strip	0	4061 lb-ft/ft	3288 lb-ft/ft	2437 lb-ft/ft	5686 lb-ft/ft
EW column strip –edge	12,517 lb-ft/ft	14,923 lb-ft/ft	25,515 lb-ft/ft	6066 lb-ft/ft	6933 lb-ft/ft