ARCHITECTURAL STRUCTURES:
FORM, BEHAVIOR, AND DESIGN
ARCH 331
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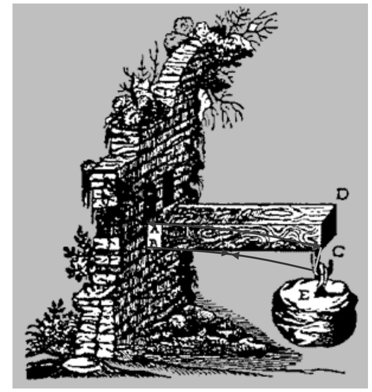
FALL 2018

lecture
ten

beams:
bending and shear stress

Beam Bending

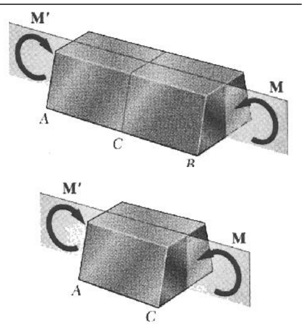
- *Galileo*
  - *relationship between stress and depth^2*
- *can see*
  - *top squishing*
  - *bottom stretching*

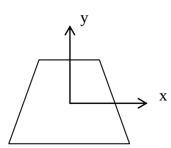


- *what are the stress across the section?*

Pure Bending

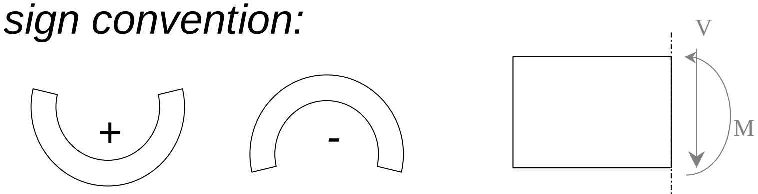
- *bending only*
- *no shear*
- *axial normal stresses from bending can be found in*
  - *homogeneous materials*
  - *plane of symmetry*
  - *follow Hooke's law*





Bending Moments

- *sign convention:*



- *size of maximum internal moment will govern our design of the section*

Normal Stresses

- *geometric fit*
  - *plane sections remain plane*
  - *stress varies linearly*

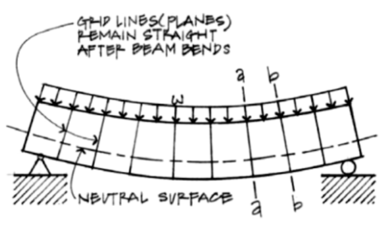


Figure 8.5(b) Beam bending under load.

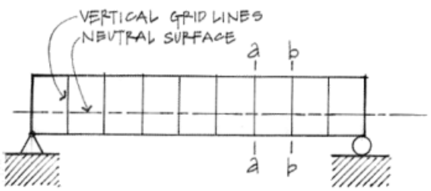
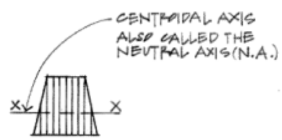


Figure 8.5(a) Beam elevation before loading.



Beam cross section.

Neutral Axis

- *stresses vary linearly*
- *zero stress occurs at the centroid*
- *<u>neutral axis</u> is line of centroids (n.a.)*

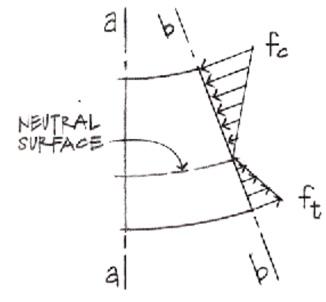


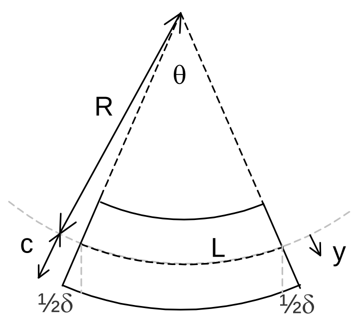
Figure 8.8 Bending stresses on section b-b.

Derivation of Stress from Strain

- *pure bending = arc shape*

$$L = R\theta$$

$$L_{outside} = (R+y)\theta$$



$$\varepsilon = \frac{\delta}{L} = \frac{L_{outside} - L}{L} = \frac{(R+y)\theta - R\theta}{R\theta} = \frac{y}{R}$$

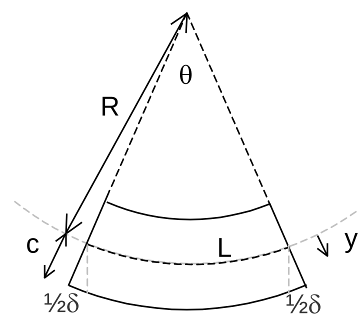
Derivation of Stress

- *zero stress at n.a.*

$$f = E\varepsilon = \frac{Ey}{R}$$

$$f_{max} = \frac{Ec}{R}$$

$$f = \frac{y}{c} f_{max}$$



Bending Moment

- *resultant moment from stresses = bending moment!*

$$M = \Sigma f y \Delta A$$

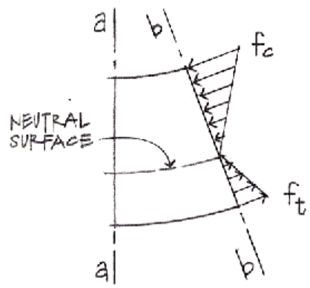


Figure 8.8 Bending stresses on section b-b.

$$= \Sigma \frac{y f_{max}}{c} y \Delta A = \frac{f_{max}}{c} \underline{\Sigma y^2 \Delta A} = \frac{f_{max}}{c} I = f_{max} S$$

Bending Stress Relations

$$\frac{1}{R} = \frac{M}{EI}$$

curvature

$$f_b = \frac{My}{I}$$

general bending stress

$$S = \frac{I}{c}$$

section modulus

$$f_b = \frac{M}{S}$$

maximum bending stress

$$S_{required} \geq \frac{M}{F_b}$$

required section modulus for design

Transverse Loading and Shear

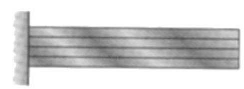
(a) Internal forces (positive shear and positive bending moment)   (b) Effect of external forces (positive shear)

- *perpendicular loading*
- *internal shear*
- *along with bending moment*

Bending vs. Shear in Design

- *bending stresses dominate*

(a)

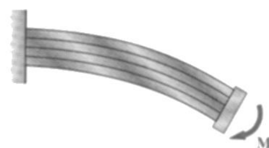


- *shear stresses exist horizontally with shear*

(b)

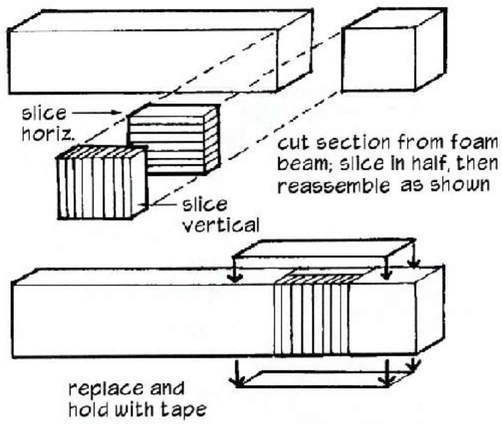
- *<u>no shear stresses</u> with pure bending*

(c)



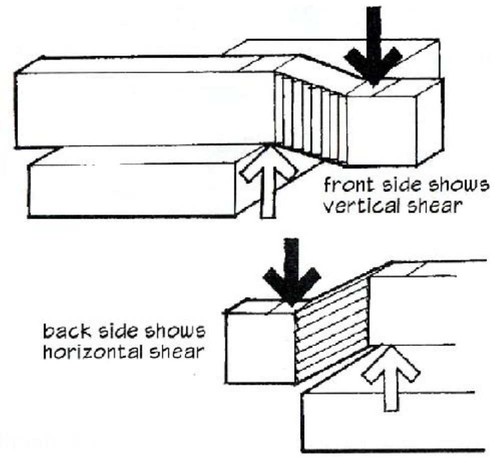
Shear Stresses

- *horizontal & vertical*



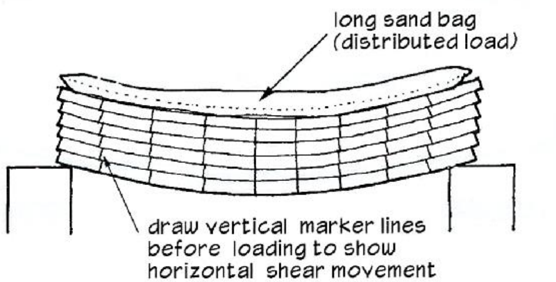
Shear Stresses

- *horizontal & vertical*



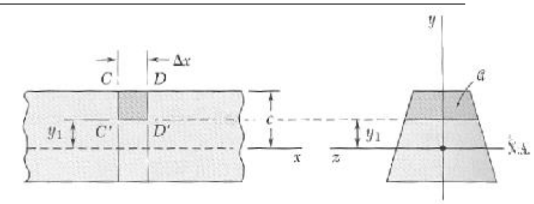
Beam Stresses

- *horizontal with bending*

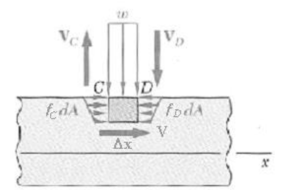


Equilibrium

- *horizontal force V needed*



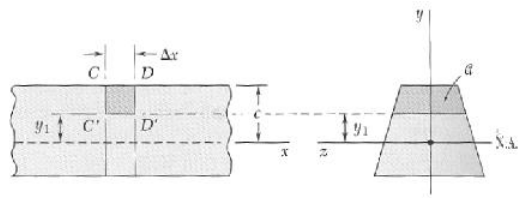
$$V_{longitudinal} = \frac{V_T Q}{I}\Delta x$$



- *Q is a moment area*

Moment of Area

- Q is a moment area with respect to the n.a. of area <u>above or below</u> the horizontal
- Q_{max} at y=0 (neutral axis)



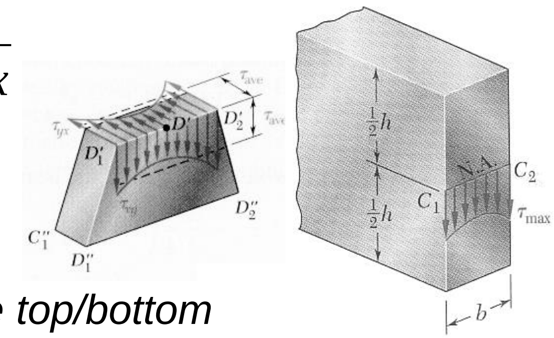
- q is shear flow:

$$q = \frac{V_{longitudinal}}{\Delta x} = \frac{V_T Q}{I}$$

Shearing Stresses

$$f_v = \frac{V}{\Delta A} = \frac{V}{b \cdot \Delta x}$$

$$f_{v-ave} = \frac{VQ}{Ib}$$

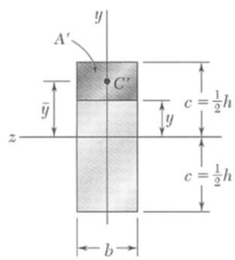


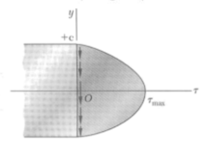
- f_{v-ave} = 0 on the top/bottom
- b min may not be with Q max
- with h/4 $\geq$ b, $f_{v\text{-}max} \leq 1.008\, f_{v\text{-}ave}$

Rectangular Sections

$$I = \frac{bh^3}{12} \qquad Q = A\bar{y} = {bh^2}/{8}$$

$$f_v = \frac{VQ}{Ib} = \frac{3V}{2A}$$

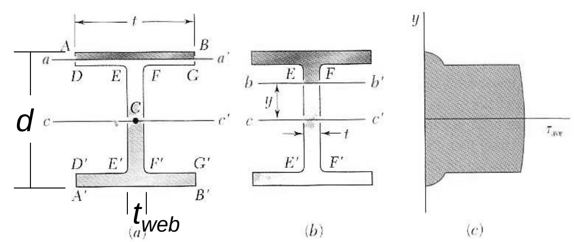




- $f_{v\text{-}max}$ occurs at <u>n.a.</u>

Steel Beam Webs

- W and S sections
  - b varies



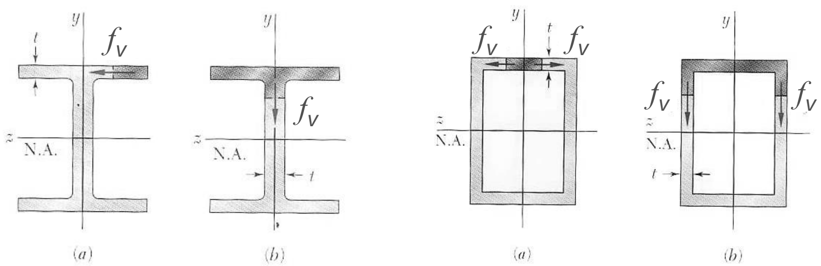
  - stress in flange negligible
  - presume constant stress in web

$$f_{v-\max} = \frac{3V}{2A} \approx \frac{V}{A_{web}}$$

Shear Flow

- loads applied in plane of symmetry
- cut made perpendicular

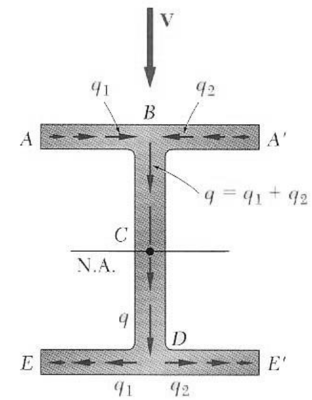
$$q = \frac{VQ}{I}$$



Shear Flow Quantity

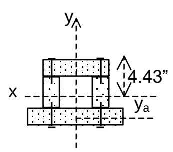
- sketch from Q

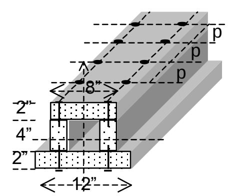
$$q = \frac{VQ}{I}$$



Connectors Resisting Shear

- plates with
  - nails
  - rivets
  - bolts
- splices





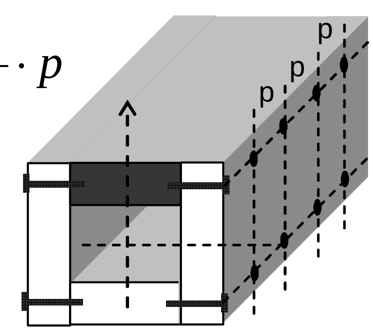
$$\frac{V_{longitudinal}}{p} = \frac{VQ}{I}$$

$$nF_{connector} \geq \frac{VQ_{connected\,area}}{I} \cdot p$$

Vertical Connectors

- isolate an area with vertical interfaces

$$nF_{connector} \geq \frac{VQ_{connected\ area}}{I} \cdot p$$



Unsymmetrical Shear or Section

- member can bend and twist
  - not symmetric
  - shear not in that plane
- shear center
  - moments balance