

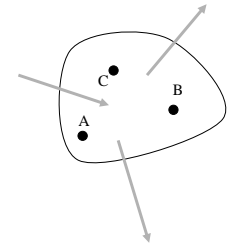
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five

# rigid body equilibrium



# Equilibrium

- rigid body
  - doesn't deform
  - coplanar force systems



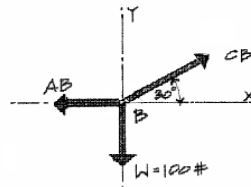
- static:  $R_x = \sum F_x = 0$

$$R_y = \sum F_y = 0$$

$$M = \sum M = 0$$

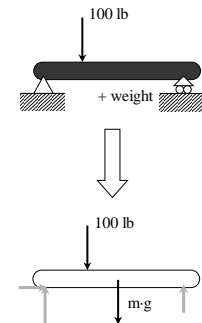
## Free Body Diagram

- FBD (sketch)
- tool to see all forces on a body or a point including
  - external forces
  - weights
  - force reactions
  - external moments
  - moment reactions
  - internal forces



## Free Body Diagram

- determine body
- FREE it from:
  - ground
  - supports & connections
- draw all external forces acting ON the body
  - reactions
  - applied forces
  - gravity

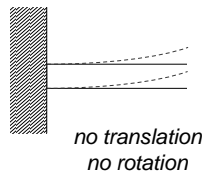
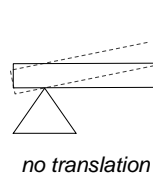
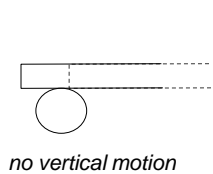


## Free Body Diagram

- sketch FBD with relevant geometry
- resolve each force into components
  - known & unknown angles – name them
  - known & unknown forces – name them
  - known & unknown moments – name them
- are any forces related to other forces?
- for the unknowns
- write only as many equilibrium equations as needed
- solve up to 3 equations

## Reactions on Rigid Bodies

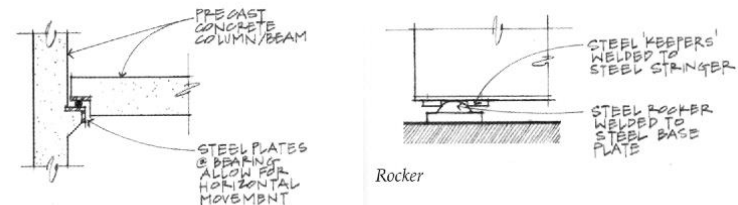
- result of applying force
- unknown size
- connection or support type
  - known direction
  - related to motion prevented



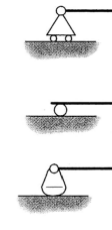
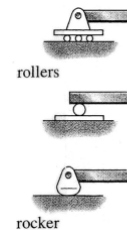
## Free Body Diagram

- solve equations
  - most times 1 unknown easily solved
  - plug into other equation(s)
- common to have unknowns of
  - force magnitudes
  - force angles
  - moment magnitudes

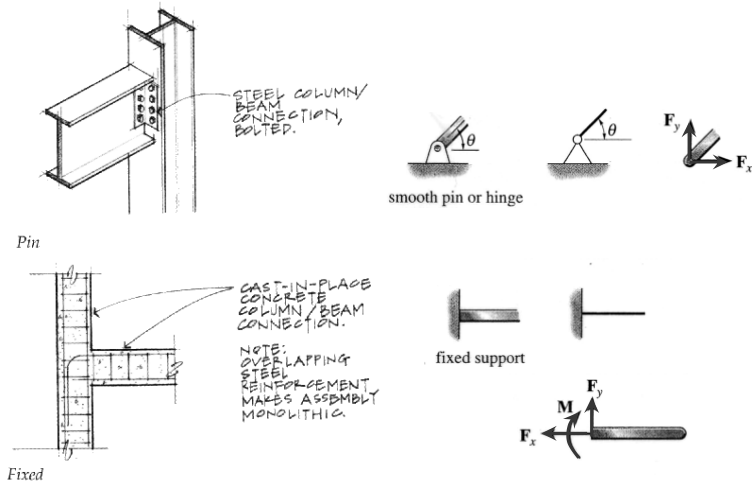
## Supports and Connections



Roller



# Supports and Connections



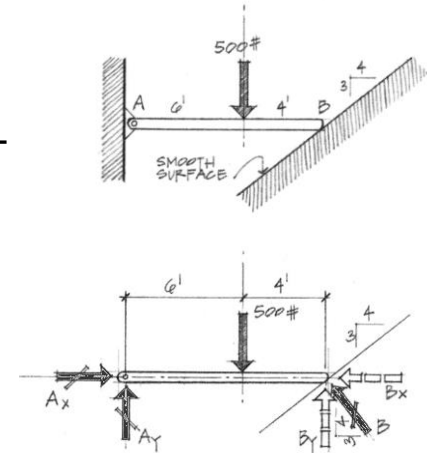
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# FBD Example

- 500 lb known
- pin –  $A_x$ ,  $A_y$
- smooth surface –  $B$  at 4:3
- 3 equations
- sum moments at
  - $A$ ?
  - $B$ ? ( $B_x$ )



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# Moment Equations

- sum moments at intersection where the most forces intersect
- multiple moment equations may not be useful
- combos:

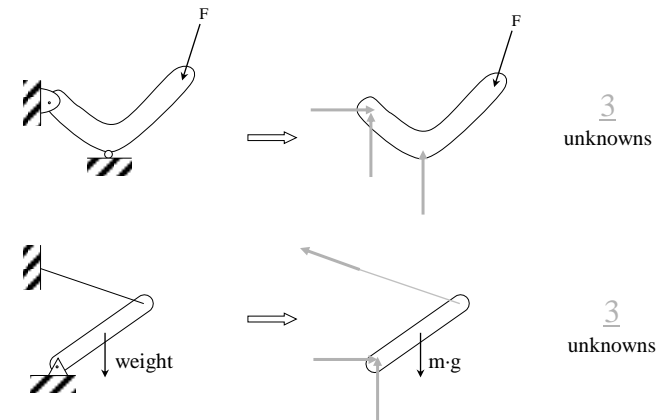
$$\begin{array}{lll}
 \sum F_x = 0 & \sum F = 0 & \sum M_1 = 0 \\
 \sum F_y = 0 & \sum M_1 = 0 & \sum M_2 = 0 \\
 \sum M_1 = 0 & \sum M_2 = 0 & \sum M_3 = 0
 \end{array}$$

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# Recognizing Reactions

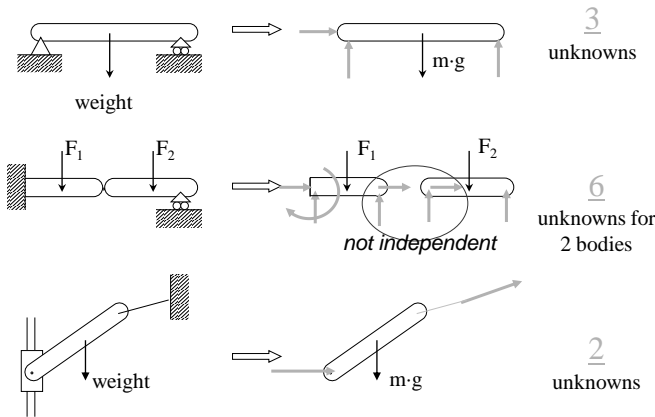


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# Recognizing Reactions



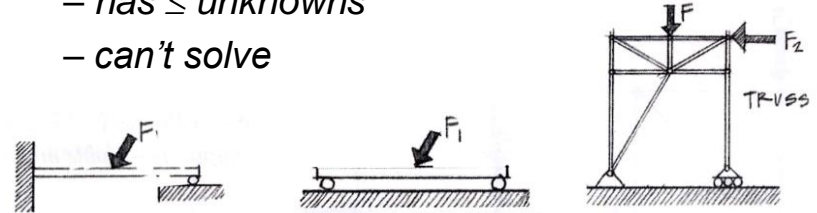
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# Constraints

- **completely constrained**
  - doesn't move
  - may not be statically determinate
- **improperly or partially constrained**
  - has  $\leq$  unknowns
  - can't solve



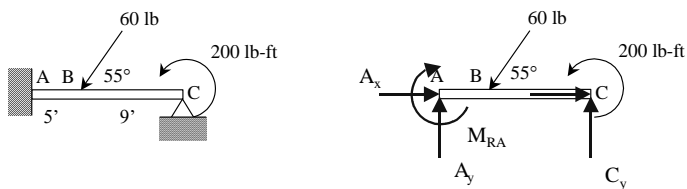
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# Constraints

- **overconstrained**
  - won't move
  - can't be solved with statics
  - statically indeterminate to  $n^{th}$  degree

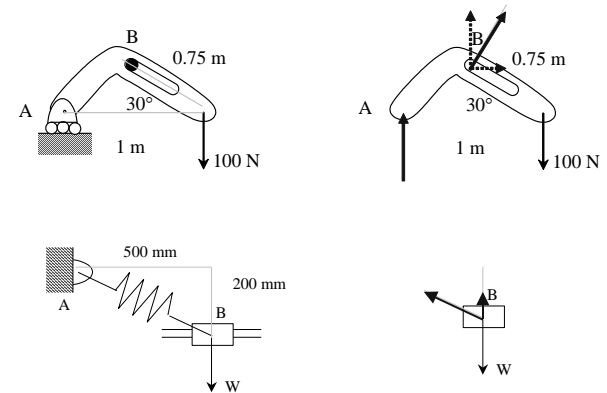


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# Partial Constraints



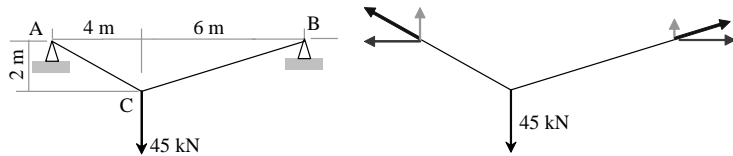
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## Cable Reactions

- **equilibrium:**
  - more reactions (4) than equations
  - but, we have slope relationships
  - x component the same everywhere



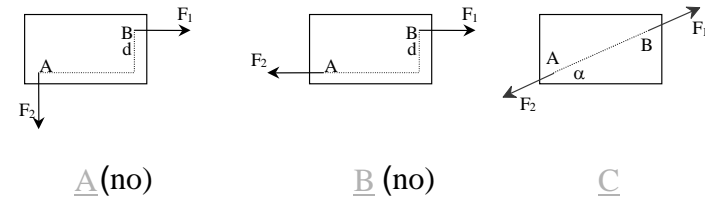
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## Two Force Rigid Bodies

- **equilibrium:**
  - forces in line, equal and opposite



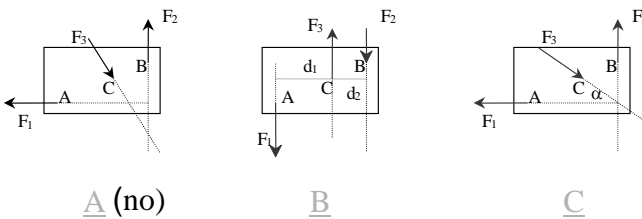
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## Three Force Rigid Bodies

- **equilibrium:**
  - concurrent or parallel forces
  - beams!

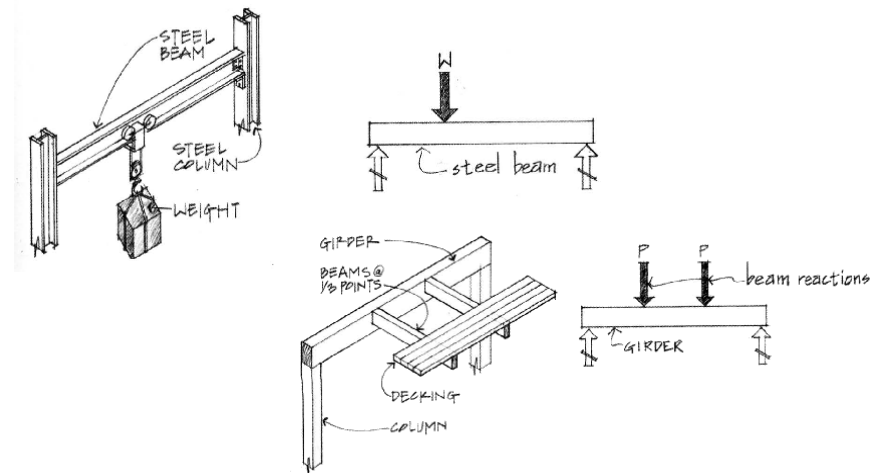


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## Concentrated Loads

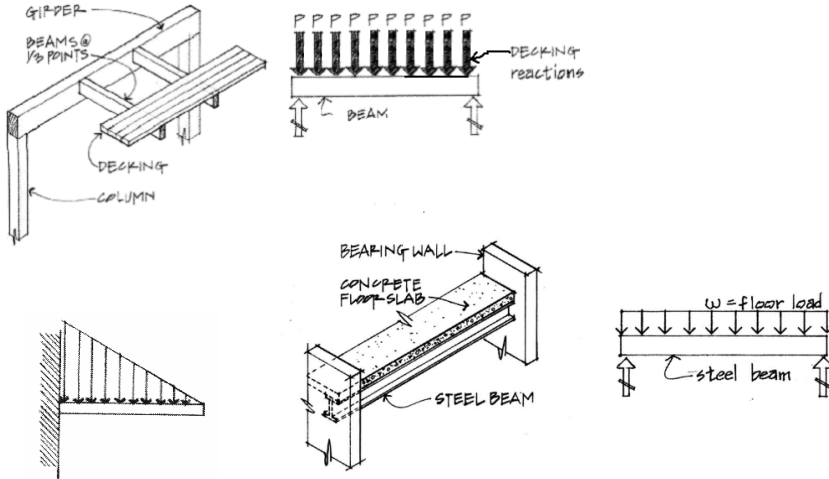


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# Distributed Loads



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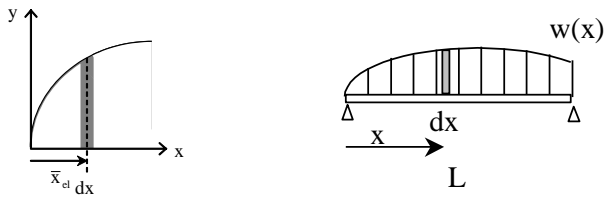
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# Equivalent Force Systems

- replace forces by resultant
- place resultant where  $M = 0$
- using calculus and area centroids

$$W = \int_0^L w dx = \int dA_{\text{loading}} = A_{\text{loading}}$$



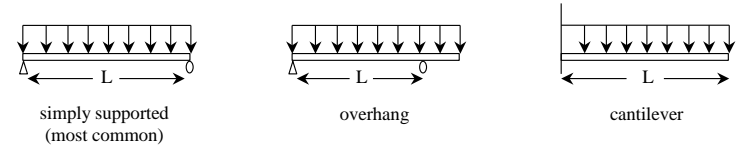
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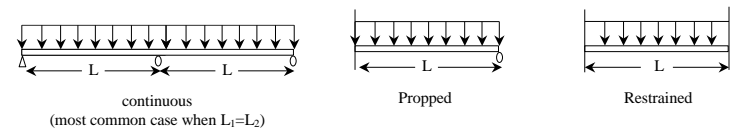
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# Beam Supports

- *statically determinate*



- *statically indeterminate*



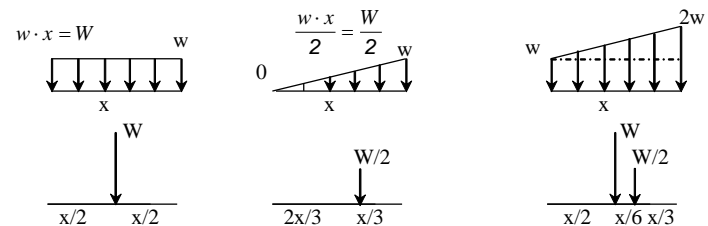
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# Load Areas

- area is width x "height" of load
- w is load per unit length
- W is total load



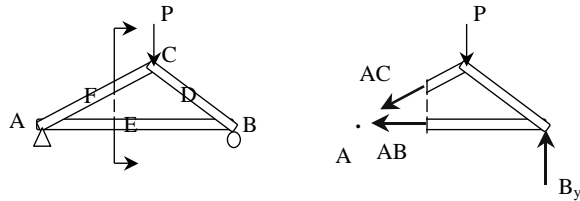
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## Method of Sections

- relies on internal forces being in equilibrium on a section
- cut to expose 3 or less members
- coplanar forces  $\rightarrow \sum M = 0$  too



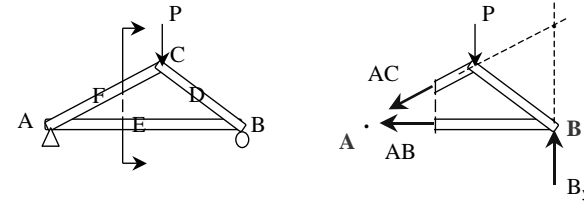
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## Method of Sections

- joints on or off the section are good to sum moments
- quick for few members
- not always obvious where to cut or sum



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