

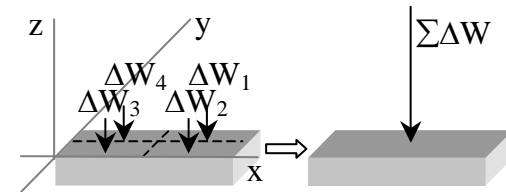
lecture  
nine



beam sections  
geometric properties

Center of Gravity

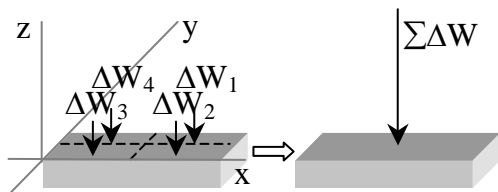
- location of equivalent weight
- determined with calculus



- sum element weights  $W = \int dW$

Center of Gravity

- “average” x & y from moment



$$\sum M_y = \sum_{i=1}^n x_i \Delta W_i = \bar{x} W \Rightarrow \bar{x} = \frac{\sum (x \Delta W)}{W}$$

“bar” means average

$$\sum M_x = \sum_{i=1}^n y_i \Delta W_i = \bar{y} W \Rightarrow \bar{y} = \frac{\sum (y \Delta W)}{W}$$

Centroid

- “average” x & y of an area
- for a volume of constant thickness
  - $\Delta W = \gamma \Delta A$  where  $\gamma$  is weight/volume
  - center of gravity = centroid of area

$$\bar{x} = \frac{\sum (x \Delta A)}{A}$$

$$\bar{y} = \frac{\sum (y \Delta A)}{A}$$



## Centroid

- for a line, sum up length

$$\bar{x} = \frac{\sum(x\Delta L)}{L}$$

$$\bar{y} = \frac{\sum(y\Delta L)}{L}$$



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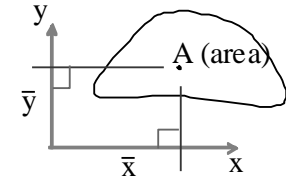
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## 1<sup>st</sup> Moment Area

- math concept
- the moment of an area about an axis

$$Q_x = \bar{y}A$$

$$Q_y = \bar{x}A$$



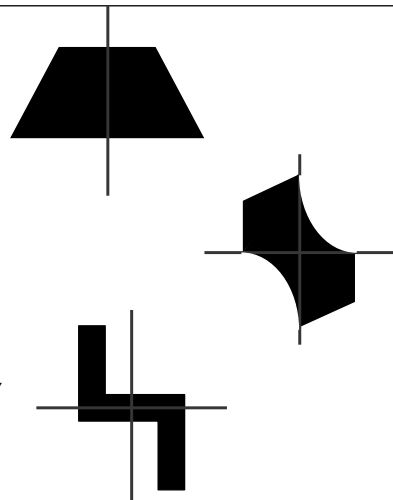
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## Symmetric Areas

- symmetric about an axis
- symmetric about a center point
- mirrored symmetry



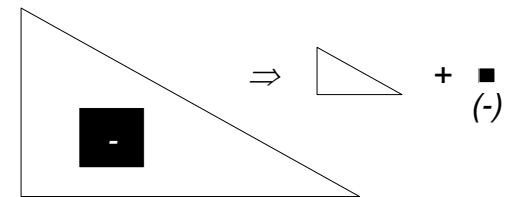
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## Composite Areas

- made up of basic shapes
- areas can be negative
- (centroids can be negative for any area)



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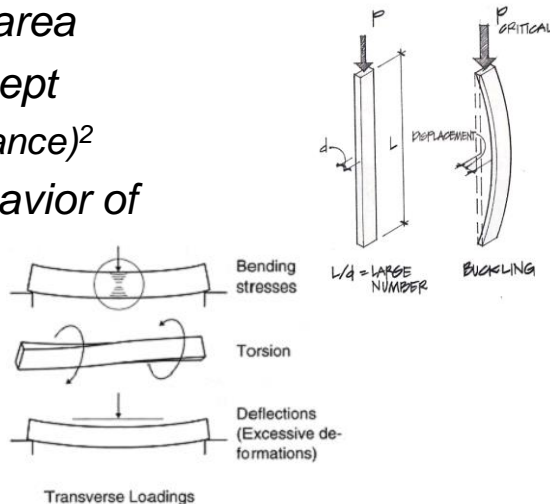
## Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table
5. Fill in table
6. Sum necessary columns
7. Calculate  $\hat{x}$  and  $\hat{y}$

Component	Area	$\bar{x}$	$\bar{x}A$	$\bar{y}$	$\bar{y}A$
$\Sigma$					

## Moments of Inertia

- 2<sup>nd</sup> moment area
  - math concept
  - area x (distance)<sup>2</sup>
- need for behavior of
  - beams
  - columns



## Area Centroids

- Table 6.1 – pg. 304

Centroids of Common Shapes of Areas and Lines

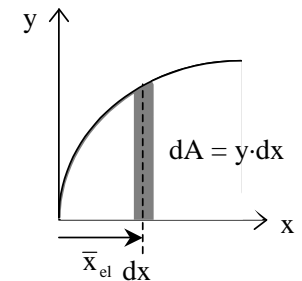
Shape	$\bar{x}$	$\bar{y}$
Triangular area 	$\frac{b}{3}$	$\frac{h}{3}$
Quarter-circular area 	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
Semicircular area 	0	$\frac{4r}{3\pi}$
Parabolic area 	$\frac{3a}{8}$	$\frac{3h}{5}$
	0	$\frac{3h}{5}$

## Moment of Inertia

- about any reference axis
- can be negative

$$I_y = \int x^2 dA$$

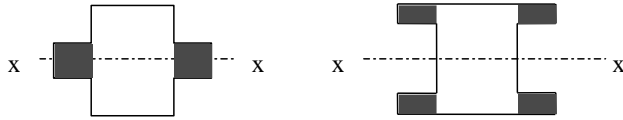
$$I_x = \int y^2 dA$$



- resistance to bending and buckling

## Moment of Inertia

- same area moved away a distance  
– larger  $I$



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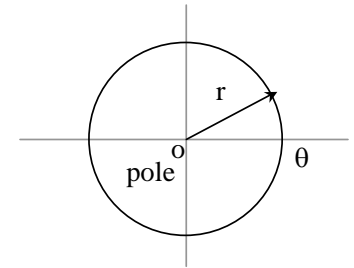
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## Polar Moment of Inertia

- for roundish shapes
- uses polar coordinates ( $r$  and  $\theta$ )
- resistance to twisting

$$J_o = \int r^2 dA$$



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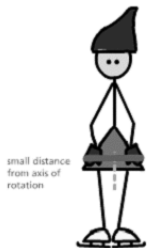
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## Radius of Gyration

- measure of inertia with respect to area

$$r_x = \sqrt{\frac{I_x}{A}}$$



When a figure skater changes position, he or she is redistributing his or her mass. Thus, every position has its own unique rotational inertia.



The rotational inertia of the figure skater increases when her arms are raised because more of her mass is redistributed further from her axis of rotation.

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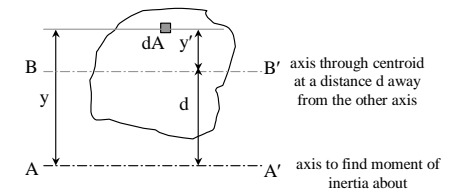
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## Parallel Axis Theorem

- can find composite  $I$  once composite centroid is known (basic shapes)

$$I_x = I_{cx} + Ad_y^2$$

$$= \bar{I}_x + Ad_y^2$$



$$I = \sum \bar{I} + \sum Ad^2$$

$$\bar{I} = I - Ad^2$$

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## Basic Procedure

1. Draw reference origin (if not given)
2. Divide into basic shapes (+/-)
3. Label shapes
4. Draw table with  $A$ ,  $\bar{x}$ ,  $\bar{x}A$ ,  $\bar{y}$ ,  $\bar{y}A$ ,  $\bar{I}$ 's,  $d$ 's, and  $Ad^2$ 's
5. Fill in table and get  $\hat{x}$  and  $\hat{y}$  for composite
6. Sum necessary columns
7. Sum  $\bar{I}$ 's and  $Ad^2$ 's

$$\begin{aligned} (d_x &= \hat{x} - \bar{x}) \\ (d_y &= \hat{y} - \bar{y}) \end{aligned}$$

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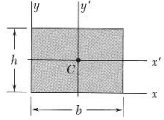
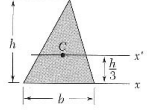
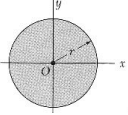
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## Area Moments of Inertia

- Table 6.2 – pg. 315: (bars refer to centroid)

- $x, y$
- $x', y'$
- $C$

Rectangle		$\bar{I}_x = \frac{1}{12}bh^3$ $\bar{I}_y = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}b^3h$ $I_y = \frac{1}{3}bh^3$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_x = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$

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