Reinforced Concrete Design

**Notation:**

- $a$ = depth of the effective compression block in a concrete beam
- $A$ = name for area
- $A_g$ = gross area, equal to the total area ignoring any reinforcement
- $A_s$ = area of steel reinforcement in concrete beam design
- $A'_s$ = area of steel compression reinforcement in concrete beam design
- $A_{st}$ = area of steel reinforcement in concrete column design
- $A_v$ = area of concrete shear stirrup reinforcement
- $ACI$ = American Concrete Institute
- $b$ = width, often cross-sectional
- $b_E$ = effective width of the flange of a concrete T beam cross section
- $b_f$ = width of the flange
- $b_w$ = width of the stem (web) of a concrete T beam cross section
- $c$ = distance from the top to the neutral axis of a concrete beam (see $x$)
- $cc$ = shorthand for clear cover
- $C$ = name for centroid
- $C_c$ = name for a compression force
- $C_s$ = compressive force in the concrete of a doubly reinforced concrete beam
- $C_{c}'$ = compressive force in the compression steel in a doubly reinforced concrete beam
- $d$ = effective depth from the top of a reinforced concrete beam to the centroid of the tensile steel
- $d'$ = effective depth from the top of a reinforced concrete beam to the centroid of the compression steel
- $d_b$ = bar diameter of a reinforcing bar
- $D$ = shorthand for dead load
- $DL$ = shorthand for dead load
- $E$ = modulus of elasticity or Young’s modulus
- $E_c$ = modulus of elasticity of concrete
- $E_s$ = modulus of elasticity of steel
- $f$ = symbol for stress
- $f_c$ = compressive stress
- $f'_c$ = concrete design compressive stress
- $f_s$ = stress in the steel reinforcement for concrete design
- $f'_s$ = compressive stress in the compression reinforcement for concrete beam design
- $f_y$ = yield stress or strength
- $f_{st}$ = yield stress or strength of transverse reinforcement
- $F$ = shorthand for fluid load
- $G$ = relative stiffness of columns to beams in a rigid connection, as is $\Psi$
- $h$ = cross-section depth
- $H$ = shorthand for lateral pressure load
- $h_f$ = depth of a flange in a T section
- $I_{\text{transformed}}$ = moment of inertia of a multi-material section transformed to one material
- $k$ = effective length factor for columns
- $l_b$ = length of beam in rigid joint
- $l_c$ = length of column in rigid joint
- $l_d$ = development length for reinforcing steel
- $l_{dh}$ = development length for hooks
- $l_n$ = clear span from face of support to face of support in concrete design
- $L$ = name for length or span length, as is $l$
- $L_r$ = shorthand for live roof load
- $LL$ = shorthand for live load
- $M_n$ = nominal flexure strength with the steel reinforcement at the yield stress and concrete at the concrete design strength for reinforced concrete beam design
- $M_o$ = maximum moment from factored loads for LRFD beam design
- $n$ = modulus of elasticity transformation coefficient for steel to concrete
\( n.a. \) = shorthand for neutral axis (N.A.)
\( pH \) = chemical alkalinity
\( P \) = name for load or axial force vector
\( P_o \) = maximum axial force with no concurrent bending moment in a reinforced concrete column
\( P_n \) = nominal column load capacity in concrete design
\( P_u \) = factored column load calculated from load factors in concrete design
\( R \) = shorthand for rain or ice load
\( R_n \) = concrete beam design ratio = \( M_u/bd^2 \)
\( s \) = spacing of stirrups in reinforced concrete beams
\( S \) = shorthand for snow load
\( t \) = name for thickness
\( T \) = name for a tension force
\( U \) = factored design value
\( V_c \) = shear force capacity in concrete
\( V_s \) = shear force capacity in steel shear stirrups
\( V_u \) = shear at a distance of \( d \) away from the face of support for reinforced concrete beam design
\( w_c \) = unit weight of concrete
\( w_{DL} \) = load per unit length on a beam from dead load
\( w_{LL} \) = load per unit length on a beam from live load
\( w_{self wt} \) = name for distributed load from self weight of member
\( w_u \) = load per unit length on a beam from load factors
\( W \) = shorthand for wind load
\( x \) = horizontal distance
\( y \) = vertical distance
\( \beta_i \) = coefficient for determining block height, \( a \), based on concrete strength, \( f'_c \)
\( \Delta \) = elastic beam deflection
\( \epsilon \) = strain
\( \epsilon_t \) = strain in the steel
\( \epsilon_y \) = strain at the yield stress
\( \lambda \) = modification factor for lightweight concrete
\( \phi \) = resistance factor
\( \phi_c \) = resistance factor for compression
\( \gamma \) = density or unit weight
\( \rho \) = radius of curvature in beam deflection relationships (see \( R \))
\( \rho_{balanced} \) = balanced reinforcement ratio in concrete beam design
\( \nu_c \) = shear strength in concrete design

**Reinforced Concrete Design**

Structural design standards for reinforced concrete are established by the *Building Code and Commentary (ACI 318-14)* published by the American Concrete Institute International, and uses strength design (also known as *limit state* design).

\( f'_c \) = concrete compressive design strength at 28 days (units of psi when used in equations)

**Materials**

Deformed reinforcing bars come in grades 40, 60 & 75 (for 40 ksi, 60 ksi and 75 ksi yield strengths). Sizes are given as # of 1/8” up
to #8 bars. For #9 and larger, the number is a nominal size (while the actual size is larger). Reinforced concrete is a composite material from a mixture of cement, coarse aggregate, fine aggregate and water, and the average density is considered to be 150 lb/ft$^3$. It has the properties that it will creep (deformation with long term load) and shrink (a result of hydration) that must be considered.

Plane sections of composite materials can still be assumed to be plane (strain is linear), but the stress distribution is not the same in both materials because the modulus of elasticity is different. ($f = E \cdot \epsilon$)

\[ f_1 = E_1 \varepsilon = - \frac{E_1 y}{R} \quad f_2 = E_2 \varepsilon = - \frac{E_2 y}{R} \]

where $R$ (or $\rho$) is the radius of curvature

In order to determine the stress, we can define $n$ as the ratio of the elastic moduli:

\[ n = \frac{E_2}{E_1} \]

$n$ is used to transform the width of the second material such that it sees the equivalent element stress.

**Transformed Section $y$ and $I$**

In order to determine stresses in all types of material in the beam, we transform the materials into a single material, and calculate the location of the neutral axis and modulus of inertia for that material.

ex: When material 1 above is concrete and material 2 is steel:

- to transform steel into concrete $n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}$
- to find the neutral axis of the equivalent concrete member we transform the width of the steel by multiplying by $n$
- to find the moment of inertia of the equivalent concrete member, $I_{\text{transformed}}$, use the new geometry resulting from transforming the width of the steel

concrete stress: $f_{\text{concrete}} = - \frac{M y}{I_{\text{transformed}} d}$

steel stress: $f_{\text{steel}} = - \frac{M y n}{I_{\text{transformed}} d}$
Reinforced Concrete Beam Members

Strength Design for Beams

Strength design method is similar to LRFD. There is a nominal strength that is reduced by a factor $\phi$ which must exceed the factored design stress. For beams, the concrete only works in compression over a rectangular “stress” block above the n.a. from elastic calculation, and the steel is exposed and reaches the yield stress, $f_y$.

For stress analysis in reinforced concrete beams
- the steel is transformed to concrete
- any concrete in tension is assumed to be cracked and to have no strength
- the steel can be in tension, and is placed in the bottom of a beam that has positive bending moment

\[ \text{Figure 8.5: Bending in a concrete beam without and with steel reinforcing.} \]
The neutral axis is where there is no stress and no strain. The concrete above the n.a. is in compression. The concrete below the n.a. (shown as x, but also sometimes named c) is considered ineffective. The steel below the n.a. is in tension. (Shown as x, but also sometimes named c.)

Because the n.a. is defined by the moment areas, we can solve for x knowing that d is the distance from the top of the concrete section to the centroid of the steel:

$$bx \cdot \frac{x}{2} - nA_s(d-x) = 0$$

x can be solved for when the equation is rearranged into the generic format with a, b & c in the binomial equation:

$$ax^2 + bx + c = 0 \quad \text{by} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

**T-sections**

If the n.a. is *above* the bottom of a flange in a T section, x is found as for a rectangular section.

If the n.a. is *below* the bottom of a flange in a T section, x is found by including the flange and the stem of the web ($b_w$) in the moment area calculation:

$$b_f h_f \left(x - \frac{h_f}{2}\right) + \left(x - h_f\right) b_w \left(x - h_f\right) - nA_s(d-x) = 0$$

**Load Combinations - (Alternative values allowed)**

1.4D  
1.2D + 1.6L + 0.5($L_r$ or $S$ or $R$)  
1.2D + 1.6($L_r$ or $S$ or $R$) + (1.0L or 0.5W)  
1.2D + 1.0W + 1.0L + 0.5($L_r$ or $S$ or $R$)  
1.2D + 1.0E + 1.0L + 0.2S  
0.9D + 1.0W  
0.9D + 1.0E

**Internal Equilibrium**

C = compression in concrete = stress x area = $0.85 f'_c a b a$

T = tension in steel = stress x area = $A_s f_y$
\[ C = T \text{ and } M_n = T(d-a/2) \]

where

- \( f'_c \) = concrete compression strength
- \( a \) = height of stress block
- \( \beta_1 \) = factor based on \( f'_c \)
- \( x \) or \( c \) = location to the neutral axis
- \( b \) = width of stress block
- \( f_y \) = steel yield strength
- \( A_s \) = area of steel reinforcement
- \( d \) = effective depth of section
  (depth to n.a. of reinforcement)

With \( C=T \), \( A_s f_y = 0.85 f'_c b a \) so \( a \) can be determined with

\[
a = \frac{A_s f_y}{0.85 f'_c b} = \beta c
\]

**Criteria for Beam Design**

For flexure design:

\[ M_u \leq \phi M_n \quad \phi = 0.9 \text{ for flexure (when the section is tension controlled)} \]

so, \( M_u \) can be set \( = \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)

**Reinforcement Ratio**

The amount of steel reinforcement is limited. Too much reinforcement, or over-reinforced will not allow the steel to yield before the concrete crushes and there is a sudden failure. A beam with the proper amount of steel to allow it to yield at failure is said to be under reinforced.

The reinforcement ratio is just a fraction: \( \rho = \frac{A_s}{b d} \) (or \( p \)) and must be less than a value determined with a concrete strain of 0.003 and tensile strain of 0.004 (minimum).

When the strain in the reinforcement is 0.005 or greater, the section is tension controlled. (For smaller strains the resistance factor reduces to 0.65 because the stress is less than the yield stress in the steel.) Previous codes limited the amount to \( 0.75 \rho_{\text{balanced}} \) where \( \rho_{\text{balanced}} \) was determined from the amount of steel that would make the concrete start to crush at the exact same time that the steel would yield based on strain \( (\varepsilon_y) \) of 0.002.

The strain in tension can be determined from \( \varepsilon_t = \frac{d-c}{c} (0.003) \). At yield, \( \varepsilon_y = \frac{f_y}{E_y} \).

The resistance factor expressions for transition and compression controlled sections are:

\[
\phi = 0.75 + (\varepsilon_t - \varepsilon_y) \frac{0.15}{(0.005 - \varepsilon_y)} \quad \text{for spiral members} \quad \text{(not less than 0.75)}
\]

\[
\phi = 0.65 + (\varepsilon_t - \varepsilon_y) \frac{0.25}{(0.005 - \varepsilon_y)} \quad \text{for other members} \quad \text{(not less than 0.65)}
\]
**Flexure Design of Reinforcement**

One method is to “wisely” estimate a height of the stress block, \( a \), and solve for \( A_s \), and calculate a new value for \( a \) using \( M_u \).

1. guess \( a \) (less than n.a.)
2. \( A_s = \frac{0.85 f'_c b a}{f_y} \)
3. solve for \( a \) from
   \[
   M_u = \phi A_s f_y (d-a/2) \:
   a = 2\left( d - \frac{M_u}{\phi A_s f_y } \right)
   \]
4. repeat from 2. until \( a \) found from step 3 matches \( a \) used in step 2.

**Design Chart Method:**
1. calculate \( R_n = \frac{M_n}{bd^2} \)
2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
3. calculate \( A_s \) and \( a \) and \( b d \)
   \[ A_s = \rho b d \quad \text{and} \quad a = \frac{A_s f_y}{0.85 f'_c b} \]

Any method can simplify the size of \( d \) using \( h = 1.1d \)

**Maximum Reinforcement**

Based on the limiting strain of 0.005 in the steel, \( x \) (or \( c \)) = 0.375\( d \) so

\( a = \beta_1 (0.375d) \) to find \( A_{s,max} \)

(\( \beta_1 \) is shown in the table above)

**Minimum Reinforcement**

Minimum reinforcement is provided even if the concrete can resist the tension. This is a means to control cracking.

Minimum required: \( A_s = \frac{3\sqrt{f'_c}}{f_y} (b_w d) \)

but not less than: \( A_s = \frac{200}{f_y} (b_w d) \)

where \( f'_c \) is in psi. This can be translated to \( \rho_{min} = \frac{3\sqrt{f'_c}}{f_y} \) but not less than \( \frac{200}{f_y} \)
Compression Reinforcement

If a section is **doubly reinforced**, it means there is steel in the beam seeing compression. The force in the compression steel at yield is equal to stress x area, \( C = A_c' F_y \). The total compression that balances the tension is now: \( T = C + C_s \). And the moment taken about the centroid of the compression stress is \( M_n = T(d-a/2) + C_s(a-d') \)

where \( A_c' \) is the area of compression reinforcement, and \( d' \) is the effective depth to the centroid of the compression reinforcement.

**T-sections (pan joists)**

T beams have an effective width, \( b_E \), that sees compression stress in a wide flange beam or joist in a slab system.

For interior T-sections, \( b_E \) is the smallest of \( L/4, b_w + 16t \), or center to center of beams.

For exterior T-sections, \( b_E \) is the smallest of \( b_w + L/12, b_w + 6t \), or \( b_w + \frac{1}{2} \) (clear distance to next beam).

When the web is in tension the minimum reinforcement required is the same as for rectangular sections with the web width \( b_w \) in place of \( b \). \( M_n = C_w (d-a/2) + C_f (d-h_f/2) \) (\( h_f \) is height of flange or \( t \)).

When the flange is in tension (negative bending), the minimum reinforcement required is the greater value of \( A_w = \frac{6 \sqrt{f_c'}}{f_y} (b_w d) \) or \( A_f = \frac{3 \sqrt{f_c'}}{f_y} (b_f d) \)

where \( f_c' \) is in psi, \( b_w \) is the beam width, and \( b_f \) is the effective flange width.

**Lightweight Concrete**

Lightweight concrete has strength properties that are different from normalweight concretes, and a modification factor, \( \lambda \), must be multiplied to the strength value of \( \sqrt{f_c'} \) for concrete for some specifications (ex. shear). Depending on the aggregate and the lightweight concrete, the value of \( \lambda \) ranges from 0.75 to 0.85, 0.85, or 0.85 to 1.0. \( \lambda \) is 1.0 for normalweight concrete.
Cover for Reinforcement

Cover of concrete over/under the reinforcement must be provided to protect the steel from corrosion. For indoor exposure, 3/4 inch is required for slabs, 1.5 inch is typical for beams, and for concrete cast against soil, 3 inches is typical.

Bar Spacing

Minimum bar spacings are specified to allow proper consolidation of concrete around the reinforcement.

Slabs

One way slabs can be designed as “one unit”-wide beams. Because they are thin, control of deflections is important, and minimum depths are specified, as is minimum reinforcement for shrinkage and crack control when not in flexure. Reinforcement is commonly small diameter bars and welded wire fabric. Maximum spacing between bars is also specified for shrinkage and crack control as five times the slab thickness not exceeding 18”. For required flexure reinforcement spacing the limit is three times the slab thickness not exceeding 18”.

Shrinkage and temperature reinforcement (and minimum for flexure reinforcement):

Minimum for slabs with grade 40 or 50 bars: \[ \rho = \frac{A_s}{bt} = 0.002 \text{ or } A_{s-min} = 0.002bt \]

Minimum for slabs with grade 60 bars: \[ \rho = \frac{A_s}{bt} = 0.0018 \text{ or } A_{s-min} = 0.0018bt \]

Shear Behavior

Horizontal shear stresses occur along with bending stresses to cause tensile stresses where the concrete cracks. Vertical reinforcement is required to bridge the cracks which are called shear stirrups.

The maximum shear for design, \( V_u \) is the value at a distance of \( d \) from the face of the support.
**Nominal Shear Strength**

The shear force that can be resisted is the shear stress × cross section area: \( V_c = \nu_c \times b_w d \)

The shear stress for beams (one way) \( \nu_c = 2\lambda \sqrt{f'_c} \) so \( \phi V_c = \phi 2\lambda \sqrt{f'_c} b_w d \)

where \( b_w \) = the beam width or the minimum width of the stem.
\( \phi = 0.75 \) for shear
\( \lambda \) = modification factor for lightweight concrete

One-way joists are allowed an increase of 10% \( V_c \) if the joists are closely spaced.

Stirrups are necessary for strength (as well as crack control): \( V_s = \frac{A_v f_{yf}}{s} \leq 8\sqrt{f'_c} b_w d (\text{max}) \)

where \( A_v \) = area of all vertical legs of stirrup
\( s \) = spacing of stirrups
\( d \) = effective depth

For shear design:

\[ V_u \leq \phi V_c + \phi V_s \quad \phi = 0.75 \text{ for shear} \]

**Spacing Requirements**

Stirrups are required when \( V_u \) is greater than \( \frac{\phi V_c}{2} \). A minimum is required because shear failure of a beam without stirrups is sudden and brittle and because the loads can vary with respect to the design values.

**Table 3-8 ACI Provisions for Shear Design**

<table>
<thead>
<tr>
<th> </th>
<th>( V_u \leq \frac{\phi V_c}{2} )</th>
<th>( \phi V_u \geq V_u &gt; \frac{\phi V_c}{2} )</th>
<th>( V_u &gt; \phi V_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Required area of stirrups, ( A_v )</strong></td>
<td>none</td>
<td>greater of ( \frac{50b_w}{s} ) and ( \frac{f_{yf}}{f'_c} b_w s )</td>
<td>( \frac{(V_u - \phi V_c)}{d} )</td>
</tr>
<tr>
<td><strong>Required Minimum</strong></td>
<td> </td>
<td> </td>
<td>( \frac{A_v f_{yf}}{60 b_w} ) or ( \frac{A_f f_{yf}}{60 b_w} )</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td> </td>
<td>( d/2 ) or 24 in.</td>
<td>( d/2 ) or 24 in. for ( (V_u - \phi V_c) \leq \phi 4\sqrt{f'_c} b_w d )</td>
</tr>
<tr>
<td><strong>Recommended Minimum</strong></td>
<td> </td>
<td> </td>
<td>( d/4 ) or 2 in. for ( (V_u - \phi V_c) &gt; \phi 4\sqrt{f'_c} b_w d )</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only; \( \phi V_u = \frac{1}{2} \lambda \sqrt{f'_c} b_w d \) \( \lambda = 0.75 \) (ACI 11.3.1.1)

**A_v = 2 × A_f for U stirrups; \( f_y \leq 60 \) ksi (ACI 11.5.2)***

†A practical limit for minimum spacing is \( d/4 \)

‡Maximum spacing based on minimum shear reinforcement at \( \frac{A_f}{50b_w} \) must also be considered (ACI 11.5.5.3).

NOTE: section numbers are pre ACI 318-14

Economical spacing of stirrups is considered to be greater than \( d/4 \). Common spacings of \( d/4 \), \( d/3 \) and \( d/2 \) are used to determine the values of \( \phi V_s \) at which the spacings can be increased.
The following figure shows that the size of \( V_n \) provided by \( V_c + V_s \) (long dashes) exceeds \( V_u/\phi \) in a step-wise function, while the spacing provided (short dashes) is at or less than the required \( s \) (limited by the maximum allowed). (Note that the maximum shear permitted from the stirrups is \( 8\sqrt{f'_c b_w d} \))

![Graph showing shear forces and stirrup spacing]

The minimum recommended spacing for the first stirrup is 2 inches from the face of the support.

**Torsional Shear Reinforcement**

On occasion beam members will see twist along the access caused by an eccentric shape supporting a load, like on an L-shaped spandrel (edge) beam. The torsion results in shearing stresses, and closed stirrups may be needed to resist the stress that the concrete cannot resist.

![Illustration of closed stirrups]

\[ A_{oh} = \text{shaded area} \]

*Fig. R11.6.3.6(b)—Definition of \( A_{oh} \)*

**Development Length for Reinforcement**

Because the design is based on the reinforcement attaining the yield stress, the reinforcement needs to be properly bonded to the concrete for a finite length (both sides) so it won’t slip. This is referred to as the development length, \( l_d \). Providing sufficient length to anchor bars that need to reach the yield stress near the end of connections are also specified by hook lengths. *Detailing reinforcement is a tedious job.* Splices are also necessary to extend the length of reinforcement that come in standard lengths. The equations are not provided here.

**Development Length in Tension**

With the proper bar to bar spacing and cover, the common development length equations are:

- **#6 bars and smaller:** \( l_d = \frac{d_h f_y}{25 \sqrt{f'_c}} \) or 12 in. minimum
- **#7 bars and larger:** \( l_d = \frac{d_h f_y}{20 \sqrt{f'_c}} \) or 12 in. minimum
Development Length in Compression

\[ l_d = \frac{d_b f_y}{50 \lambda f'_c} \leq 0.0003 f_y d_b \text{ or } 8 \text{ in. minimum} \]

Hook Bends and Extensions

The minimum hook length is \( l_{dh} = \frac{d_b f_y}{50 \lambda f'_c} \) but not less than the larger of \( 8d_b \) and 6 in.

\[ \text{Figure 9-17: Minimum requirements for } 90^\circ \text{bar hooks.} \]

\[ \text{Figure 9-18: Minimum requirements for } 180^\circ \text{bar hooks.} \]

Modulus of Elasticity & Deflection

\( E_c \) for deflection calculations can be used with the transformed section modulus in the elastic range. After that, the cracked section modulus is calculated and \( E_c \) is adjusted.

Code values:

\[ E_c = 57,000 \sqrt{f'_c} \text{ (normal weight)} \quad E_c = w_c^{1.5} 33 \sqrt{f'_c}, \quad w_c = 90 \text{ lb/ft}^3 - 160 \text{ lb/ft}^3 \]

Deflections of beams and one-way slabs need not be computed if the overall member thickness meets the minimum specified by the code, and are shown in 7.3.1.1 (see Slabs). The span lengths for continuous beams or slabs is taken as the clear span, \( l_n \).

Criteria for Flat Slab & Plate System Design

Systems with slabs and supporting beams, joists or columns typically have multiple bays. The horizontal elements can act as one-way or two-way systems. Most often the flexure resisting elements are continuous, having positive and negative bending moments. These moment and shear values can be found using beam tables, or from code specified approximate design factors. Flat slab two-way systems have drop panels (for shear), while flat plates do not.

Two way shear at columns is resisted by the thickness of the slab at a perimeter of \( d/2 \) away from the face of the support by the shear stress \( \times \) cross section area: \( V_c = v_c \times b_o d \)
The shear stress (two way) \( \tau = 4 \lambda \sqrt{f'_c} \) so \( \phi \tau = \phi 4 \lambda \sqrt{f'_c} b_o d \)

where \( b_o = \) perimeter length.
\( \phi = 0.75 \) for shear  
\( \lambda = \) modification factor for lightweight concrete

**Criteria for Column Design**

(American Concrete Institute) ACI 318-14 Code and Commentary:

\[ P_u \leq \phi P_n \]

where
\( P_u = \) a factored load  
\( \phi = \) a resistance factor  
\( P_n = \) the nominal load capacity (strength)

Load combinations, ex:  
1.4D (D is dead load)  
1.2D + 1.6L (L is live load)  
1.2D + 1.6Lr + 0.5W (W is wind load)  
0.90D + 1.0W

For compression, \( \phi_c = 0.75 \) and \( P_n = 0.85 P_o \) for spirally reinforced,  
\( \phi_c = 0.65 \) \( P_n = 0.8 P_o \) for tied columns where \( P_o = 0.85 f'_c (A_g - A_{st}) + f_s A_{st} \) and \( P_o \)

is the name of the maximum axial force with no concurrent bending moment.

Columns which have reinforcement ratios, \( \rho = \frac{A_{st}}{A_g} \), in the range of 1% to 2% will usually be the most economical, with 1% as a minimum and 8% as a maximum by code.

Bars are symmetrically placed, typically.

**Columns with Bending (Beam-Columns)**

Concrete columns rarely see only axial force and must be designed for the combined effects of axial load and bending moment.

The *interaction* diagram shows the reduction in axial load a column can carry with a bending moment.

Design aids commonly present the interaction diagrams in the form of load vs. equivalent eccentricity for standard column sizes and bars used.
**Eccentric Design**

The strength interaction diagram is dependent upon the strain developed in the steel reinforcement.

If the strain in the steel is less than the yield stress, the section is said to be *compression controlled*.

Below the *transition zone*, where the steel starts to yield, and when the net tensile strain in the reinforcement exceeds 0.005 the section is said to be *tension controlled*. This is a ductile condition and is preferred.

**Rigid Frames**

Monolithically cast frames with beams and column elements will have members with shear, bending and axial loads. Because the joints can rotate, the effective length must be determined from methods like that presented in the handout on Rigid Frames. The charts for evaluating $k$ for non-sway and sway frames can be found in the ACI code.

**Frame Columns**

Because joints can rotate in frames, the effective length of the column in a frame is harder to determine. The stiffness $(EI/L)$ of each member in a joint determines how rigid or flexible it is. To find $k$, the relative stiffness, $G$ or $\Psi$, must be found for both ends, plotted on the alignment charts, and connected by a line for braced and unbraced frames.

\[
G = \Psi = \frac{\sum EI}{l_c} / \frac{\sum EI}{l_b}
\]

where

- $E$ = modulus of elasticity for a member
- $I$ = moment of inertia of a member
- $l_c$ = length of the column from center to center
- $l_b$ = length of the beam from center to center

- For pinned connections we typically use a value of 10 for $\Psi$.
- For fixed connections we typically use a value of 1 for $\Psi$. 
Braced – non-sway frame

Unbraced – sway frame

(a) Nonsway Frames

(b) Sway Frames
**Factored Moment Resistance of Concrete Beams, \( \phi M_n \) (k-ft) with \( f'c = 4 \text{ ksi}, f_y = 60 \text{ ksi} \)**

### Approximate Values for \( a/d \)

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<th>( b \times d ) (in)</th>
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<th>0.3</th>
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*Table yields values of factored moment resistance in kip-ft with reinforcement indicated. Reinforcement choices shown in parentheses require greater width of beam or use of two stack layers of bars. (Adapted and corrected from Simplified Engineering for Architects and Builders, 11th ed, Ambrose and Tripeny, 2010.*
Column Interaction Diagrams
Column Interaction Diagrams

FIGURE D.21 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.22 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.23 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)

FIGURE D.24 Column interaction diagram for circular spiral column. (Courtesy of the American Concrete Institute, Farmington Hills, MI.)
Beam / One-Way Slab Design Flow Chart

Collect data: L, ω, γ, Δlimits, hmin; find beam charts for load cases and Δactual equations (self weight = area x density)

Collect data: load factors, f_y, f'_c

Find V's & M_u from constructing diagrams or using beam chart formulas with the factored loads (V_u-max is at d away from face of support)

Determine M_n required by M_u/φ, choose method

Chart (R_n vs ρ)

Select ρ_min ≤ ρ ≤ ρ_max

Find R_n off chart with f_y, f'_c and select ρ_min ≤ ρ ≤ ρ_max

Choose b & d combination based on R_n and h_min (slabs), estimate h with 1" bars (#8)

Calculate A_s = ρbd

Select bar size and spacing to fit width or 12 in strip of slab and not exceed limits for crack control

Calculate a, φM_u

Is M_u ≤ φM_n?

Increase h, find d

NO

Increase h, find d*

Is ρ_min ≤ ρ ≤ ρ_max?

YES

YES

ON to shear reinforcement for beams

NO
Beam, Adequate for Flexure

Determine shear capacity of plain concrete based on $f'_c$, $b$ & $d$, $\phi V_c$

Is $V_u$ (at $d$ for beams) $\leq \phi V_c$?

Beam? NO

YES

Determine $\phi V_s = (V_u - \phi V_c)$

Is $\phi V_s \leq \frac{b}{2} \sqrt{f'_c b d}$?

YES

Determine $s$ & $A_v$

Find where $V = \phi V_c$
and provide minimum $A_v$ and change $s$

Find where $V = \frac{1}{2} \phi V_c$
and provide stirrups just past that point

Beam? NO

Increase $h$ and re-evaluate flexure ($A_v$ and $\phi M_f$ of previous page)*

NO

YES

Is $V_u < \frac{1}{2} \phi V_c$?

NO

YES

Beam / One-Way Slab Design Flow Chart - continued