1. SIMPLE BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = \frac{wl}{2} \)

\[ R = V = \frac{wl}{2} \]

\[ V_x = \frac{wl}{2} \left( \frac{l}{2} - x \right) \]

\[ M_{\text{max.}} \text{ (at center)} = \frac{wx^2}{8} \]

\[ M_x = \frac{wx}{2} \left( l - x \right) \]

\[ \Delta_{\text{max.}} \text{ (at center)} = \frac{5wx^4}{384EI} \]

\[ \Delta_x = \frac{wx}{24EI} \left( \frac{l}{2} - 2x^2 + x^4 \right) \]

2. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO ONE END

Total Equiv. Uniform Load \( = \frac{16W}{9} \frac{l}{3} = 1.0264W \)

\[ R_1 = V_1 = \frac{W}{3} \]

\[ R_2 = V_2 \text{ max.} = \frac{2W}{3} \]

\[ V_x = \frac{W}{3} - \frac{Wx^2}{l} \]

\[ M_{\text{max.}} \text{ (at } x = \frac{l}{3} \text{ )} = \frac{5.774l}{9} \]

\[ M_x = \frac{W}{3} \left( \frac{l}{2} - x^2 \right) \]

\[ \Delta_{\text{max.}} \text{ (at } x = \sqrt{\frac{8}{15}} \text{ )} = 0.1304 \frac{Wl^3}{EI} \]

\[ \Delta_x = \frac{Wx}{180EI \frac{l}{2}} \left( 3x^4 - 10x^2 + 7 \right) \]

3. SIMPLE BEAM—LOAD INCREASING UNIFORMLY TO CENTER

Total Equiv. Uniform Load \( = \frac{4W}{3} \)

\[ R = V = \frac{W}{2} \]

\[ V_x = \frac{W}{2} \left( \frac{l}{4} - 4x^3 \right) \]

\[ M_{\text{max.}} \text{ (at center)} = \frac{Wl}{6} \]

\[ M_x = \frac{Wx}{2} \left( \frac{1}{2} - \frac{2x^2}{3l^2} \right) \]

\[ \Delta_{\text{max.}} \text{ (at center)} = \frac{Wl^3}{60EI} \]

\[ \Delta_x = \frac{Wx}{240EI \frac{l}{2}} \left( 5l^2 - 4x^2 \right) \]

4. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED

\[ R_1 = V_1 \text{ (max. when } a < c \text{) } = \frac{wb}{2l} \left( 2c + b \right) \]

\[ R_2 = V_2 \text{ (max. when } a > c \text{) } = \frac{wb}{2l} \left( 2c + b \right) \]

\[ V_x \text{ (when } x > a \text{ and } c > a \text{) } = R_1 - \frac{w}{c} (x - a) \]

\[ M_{\text{max.}} \text{ at } x = a + \frac{R_1}{w} \]

\[ M_x \text{ (when } x < a \text{) } = R_1x - \frac{w}{c} \left( x - a \right) \]

\[ M_x \text{ (when } x > a \text{ and } c > a \text{) } = R_1x - \frac{w}{c} \left( x - a \right)^2 \]

5. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT ONE END

\[ R_1 = V_1 \text{ max. } = \frac{wa}{2l} \left( 2l - a \right) \]

\[ R_2 = V_2 \]

\[ V_x \text{ (when } x < a \text{) } = R_1 - \frac{wa}{l} \]

\[ M_{\text{max.}} \text{ at } x = \frac{R_1}{w} \]

\[ M_x \text{ (when } x < a \text{) } = \frac{w}{2l} \left( x^2 - a^2 \right) \]

\[ M_x \text{ (when } x > a \text{) } = R_1x - \frac{wa}{l} \left( x - a \right)^2 \]

\[ \Delta_x \text{ (when } x < a \text{) } = \frac{wa^2}{24EI} \left( x^3 - a^3 \right) \]

\[ \Delta_x \text{ (when } x > a \text{) } = \frac{wa^3}{24EI} \left( \frac{a}{x} - 3ax^2 + 3ax^3 - x^4 \right) \]

6. SIMPLE BEAM—UNIFORM LOAD PARTIALLY DISTRIBUTED AT EACH END

\[ R_1 = V_1 \]

\[ R_2 = V_2 \text{ (when } x < a \text{) } = \frac{wa}{2l} \left( 2l - a \right) \]

\[ V_x \text{ (when } x > a \text{ and } a + b \text{) } = R_1 - \frac{wa}{l} \]

\[ M_{\text{max.}} \text{ at } x = l - R_1 \text{ when } R_1 < \frac{wa}{l} \]

\[ M_x \text{ (when } x < a \text{) } = \frac{wa}{2l} \left( 2x - a \right) \]

\[ M_x \text{ (when } x > a \text{ and } b \text{) } = \frac{wa}{2l} \left( x - a \right)^2 \]
7. **SIMPLE BEAM—CONCENTRATED LOAD AT CENTER**

Total Equiv. Uniform Load \[ = 2P \]

\[ R = V = \frac{P}{2} \]

\[ M_{\text{max}} \text{ (at point of load)} = \frac{P}{4} \]

\[ M_x \text{ (when } x < \frac{1}{2} \text{)} = \frac{P}{2}x \]

\[ \Delta_{\text{max}} \text{ (at point of load)} = \frac{P^2a}{48EI} \]

\[ \Delta_x \text{ (when } x < \frac{1}{2} \text{)} = \frac{P}{48EI} (3/8 - 4x^3) \]

8. **SIMPLE BEAM—CONCENTRATED LOAD AT ANY POINT**

Total Equiv. Uniform Load \[ = \frac{8Pab}{l} \]

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{Pa}{l} \]

\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{Pb}{l} \]

\[ M_{\text{max}} \text{ (at point of load)} = \frac{Pab}{l} \]

\[ M_x \text{ (when } x < a \text{)} = \frac{Pb}{3EI} \]

\[ \Delta_{\text{max}} \text{ (at } x = \frac{a}{3} \text{ when } a > b \text{)} = \frac{Pab^2}{27EI \cdot l} \]

\[ \Delta_x \text{ (at point of load)} = \frac{Pa^2b^2}{3EI \cdot l} \]

\[ \alpha \text{ (when } x < a \text{)} = \frac{Pb}{6EI} (\frac{1}{2} - b^2 - x^2) \]

9. **SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS SYMMETRICALLY PLACED**

Total Equiv. Uniform Load \[ = \frac{8P}{l} \]

\[ R = V = \frac{P}{l} \]

\[ M_{\text{max}} \text{ (between loads)} = \frac{P}{l} \]

\[ M_x \text{ (when } x < a \text{)} = \frac{P}{6EI} \]

\[ \Delta_{\text{max}} \text{ (at center)} = \frac{Pa}{24EI} (\frac{3}{8} - 4a^3) \]

\[ \Delta_x \text{ (when } x < a \text{)} = \frac{P}{6EI} (3a - 3a^2 - x^2) \]

\[ \Delta_x \text{ (when } x > a \text{ and } < (l - a) \text{)} = \frac{P}{6EI} (3a - 3a^2 - x^2) \]

10. **SIMPLE BEAM—TWO EQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED**

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{P}{l} (l - a + b) \]

\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{P}{l} (l - b + a) \]

\[ V_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = \frac{P}{l} (b - a) \]

\[ M_1 \text{ (max. when } a < b \text{)} = -R_1a \]

\[ M_2 \text{ (max. when } a < b \text{)} = -R_2b \]

\[ M_x \text{ (when } x < a \text{)} = -R_1x \]

\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1x - P(x - a) \]

11. **SIMPLE BEAM—TWO UNEQUAL CONCENTRATED LOADS UNSYMMETRICALLY PLACED**

\[ R_1 = V_1 \]

\[ R_2 = V_2 \]

\[ V_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = \frac{P}{l} (b - a) \]

\[ M_1 \text{ (max. when } R_1 < P_1 \text{)} = -R_1a \]

\[ M_2 \text{ (max. when } R_2 < P_2 \text{)} = -R_2b \]

\[ M_x \text{ (when } x < a \text{)} = -R_1x \]

\[ M_x \text{ (when } x > a \text{ and } < (l - b) \text{)} = R_1x - P(x - a) \]

12. **BEAM FIXED AT ONE END, SUPPORTED AT OTHER—UNIFORMLY DISTRIBUTED LOAD**

Total Equiv. Uniform Load \[ = w \frac{l}{12} \]

\[ R_1 = V_1 = \frac{3w}{8}l \]

\[ R_2 = V_2 = \frac{5w}{8}l \]

\[ V_x = \frac{w}{8} \]

\[ M_{\text{max}} \text{ (at } x = \frac{3}{8}l \text{)} = \frac{9}{128}wl^3 \]

\[ M_{\text{max}} \text{ (at } x = \frac{1}{16} (1 + \sqrt{33}) = .4215l \text{)} = \frac{w^4}{185EI} \]

\[ M_{\text{max}} \text{ (at } x = \frac{1}{16} (1 + \sqrt{33}) = .4215l \text{)} = \frac{w^4}{48EI} (l^2 - 3lx^2 + 2x^3) \]
13. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load \( \frac{3P}{2} \)

\[ R_1 = V_1 = \frac{5P}{16} \]

\[ R_2 = V_2 \text{ max.} = \frac{11P}{16} \]

\( M_{\text{max.}} \text{ (at fixed end)} = \frac{3PL}{16} \)

\( M_1 \text{ (at point of load)} = \frac{3PL}{32} \)

\( M_2 \text{ (when } x < \frac{1}{2} \text{)} = \frac{3PL}{16} \)

\( M_2 \text{ (when } x > \frac{1}{2} \text{)} = \frac{P}{2} - \frac{11x}{16} \)

\( \Delta_{\text{max.}} \text{ (at } x = \frac{1}{5} - \frac{447}{240} \text{)} = \frac{9P^2}{48EI} \text{ } \frac{1}{5} - \frac{0.00637 \cdot P^2}{EI} \)

\( \Delta_x \text{ (at point of load)} = \frac{7P^2}{768EI} \)

\( \Delta_x \text{ (when } x < \frac{1}{2} \text{)} = \frac{96EI}{(3/4 - 5x^2)} \)

\( \Delta_x \text{ (when } x > \frac{1}{2} \text{)} = \frac{P}{96EI} (x - \frac{1}{2})(11x - 2) \)

14. BEAM FIXED AT ONE END, SUPPORTED AT OTHER—CONCENTRATED LOAD AT ANY POINT

\[ R_1 = V_1 = \frac{Pa}{2l^2} (a + l) \]

\[ R_2 = V_2 = \frac{Pa}{2l^2} (3l^2 - a^2) \]

\( M_1 \text{ (at point of load)} = R_1a \)

\( M_2 \text{ (at fixed end)} = R_2a \)

\( M_2 \text{ (when } x < a \text{)} = R_1x - P\left(x - a\right) \)

\( M_2 \text{ (when } x > a \text{)} = \frac{Pa}{2l^2} a \)

\( \Delta_{\text{max.}} \text{ (when } a < \frac{4}{144} \text{ at } x = \frac{1}{2} - \frac{1}{2} \text{)} = \frac{Pa}{96EI} (\frac{1}{2} - \frac{1}{2})^3 \)

\( \Delta_{\text{max.}} \text{ (when } a > \frac{4}{144} \text{ at } x = \frac{1}{2} + \frac{1}{2} \text{)} = \frac{Pab^2}{8EI} \left(\frac{a}{2}+a\right) \)

\( \Delta_a \text{ (at point of load)} = \frac{Pa^2b^2}{12EI^2} (3l^2 - a^2) \)

\( \Delta_x \text{ (when } x < a \text{)} = \frac{Pa}{12EI} \left(\frac{1}{2} - \frac{1}{2}\right) (3l^2 - 2l^2 - 2ax^2) \)

\( \Delta_x \text{ (when } x > a \text{)} = \frac{2Pb^2x}{12EI^2} \left(3l^2 - 2l^2 - 2ax^2\right) \)

15. BEAM FIXED AT BOTH ENDS—UNIFORMLY DISTRIBUTED LOADS

Total Equiv. Uniform Load \( \frac{2P}{3} \)

\[ R = V = \frac{P}{2} \text{ at } x = \frac{l}{2} \]

\[ M_{\text{max.}} \text{ (at ends)} = \frac{P}{l} \]

\( M_1 \text{ (at center)} = \frac{P}{24} \)

\( M_x \text{ (at center)} = \frac{P}{24} \left(4l^2 - 4x^2\right) \)

\( \Delta_{\text{max.}} \text{ (at center)} = \frac{9P^2}{8EI} \)

\( \Delta_x \text{ (at center)} = \frac{P^2}{24EI} \left(1 - x^2\right) \)

16. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT CENTER

Total Equiv. Uniform Load \( P \)

\[ R = V = \frac{P}{2} \text{ at } x = \frac{l}{2} \]

\( M_{\text{max.}} \text{ (at center and ends)} = \frac{P}{l} \)

\( M_x \text{ (when } x < \frac{1}{2} \text{)} = \frac{P}{8} \left(4x^2 - l\right) \)

\( M_x \text{ (when } x > \frac{1}{2} \text{)} = \frac{P}{96EI} \left(x - \frac{1}{2}\right) (11x - 2) \)

\( \Delta_{\text{max.}} \text{ (at center)} = \frac{9P^2}{8EI} \)

\( \Delta_x \text{ (when } x < \frac{1}{2} \text{)} = \frac{P^2}{48EI} \left(3l - 4x\right) \)

17. BEAM FIXED AT BOTH ENDS—CONCENTRATED LOAD AT ANY POINT

\[ R_1 = V_1 \text{ (max. when } a < b \text{)} = \frac{Pa}{1} \left(3l^2 + a^2\right) \]

\[ R_2 = V_2 \text{ (max. when } a > b \text{)} = \frac{Pa}{1} \left(a + 3l^2\right) \]

\( M_1 \text{ (max. when } a < b \text{)} = \frac{Pa^2b^2}{12EI^2} (3l^2 - a^2) \)

\( M_2 \text{ (max. when } a > b \text{)} = \frac{Pa^2b^2}{12EI^2} (3l^2 - a^2) \)

\( M_a \text{ (at point of load)} = \frac{Pab}{1} \left(l - \frac{a}{l}\right) \)

\( M_1 \text{ (at point of load)} = \frac{Pab}{1} \left(l - \frac{a}{l}\right) \)

\( \Delta_{\text{max.}} \text{ (when } a > b \text{ at } x = \frac{3a}{2} \) = \frac{2Pb^2a}{3EI} \left(3a^2 - a^2 - bx\right) \)

\( \Delta_a \text{ (at point of load)} = \frac{Pb^2a^2}{6EI^2} \left(3a^2 - 3ax - bx\right) \)

\( \Delta_x \text{ (when } x < a \text{)} = \frac{2Pb^2a}{12EI} \left(3a^2 - a^2 - bx\right) \)
18. CANTILEVER BEAM—LOAD INCREASING UNIFORMLY TO FIXED END

Total Equiv. Uniform Load \( = \frac{8}{3} W \)

\[ W = \frac{wl}{2} \]

19. CANTILEVER BEAM—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = 4wl \)

\[ W = \frac{wl}{2} \]

20. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—UNIFORMLY DISTRIBUTED LOAD

Total Equiv. Uniform Load \( = \frac{8}{3} wl \)

\[ W = \frac{wl}{2} \]

21. CANTILEVER BEAM—CONCENTRATED LOAD AT ANY POINT

Total Equiv. Uniform Load \( = \frac{8Pb}{l} \)

\[ R = V \]

\[ M_{\text{max}} \text{ (at fixed end)} = Pb \]

\[ M_{x} \text{ (when } x > a \text{)} = P(x-a) \]

\[ \Delta_{\text{max}} \text{ (at free end)} = \frac{Pb}{3EI} (3l - b) \]

\[ \Delta_{a} \text{ (at point of load)} = \frac{Pb}{3EI} \]

\[ \Delta_{x} \text{ (when } x < a \text{)} = \frac{Pb}{6EI} (3l - 3x - b) \]

\[ \Delta_{x} \text{ (when } x > a \text{)} = \frac{P(l-x)^{2}}{6EI} (3b - l + x) \]

22. CANTILEVER BEAM—CONCENTRATED LOAD AT FREE END

Total Equiv. Uniform Load \( = 8P \)

\[ R = V \]

\[ M_{\text{max}} \text{ (at fixed end)} = P/I \]

\[ M_{x} \text{ (at fixed end)} = P/I \]

\[ \Delta_{\text{max}} \text{ (at free end)} = \frac{P}{3EI} \]

\[ \Delta_{x} \text{ (at free end)} = \frac{P}{6EI} (2/l - 3x + x^{3}) \]

23. BEAM FIXED AT ONE END, FREE TO DEFLECT VERTICALLY BUT NOT ROTATE AT OTHER—CONCENTRATED LOAD AT DEFLECTED END

Total Equiv. Uniform Load \( = 4P \)

\[ R = V \]

\[ M_{\text{max}} \text{ (at both ends)} = \frac{P}{2} \]

\[ M_{x} \text{ (at both ends)} = \frac{P}{2} \]

\[ \Delta_{\text{max}} \text{ (at deflected end)} = \frac{P}{12EI} \]

\[ \Delta_{x} \text{ (at deflected end)} = \frac{P(l-x)^{2}}{12EI} (l+2x) \]
24. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD

\[ R_1 = V_1 = \frac{wa^2}{2l} \]

\[ R_2 = V_2 + V_3 = \frac{wa}{2} \]

\[ V_x \quad \text{(between supports)} = R_x = -ax \]

\[ V_{x1} \quad \text{(for overhang)} = w (a - x_1) \]

\[ M_{x1} \quad \text{(between supports)} = \frac{wx_1a}{2l} \]

\[ M_{x1} \quad \text{(for overhang)} = \frac{wx_1a}{2l} \]

\[ \Delta x \quad \text{(between supports)} = \frac{ax_1^2}{24EI} \]

\[ \Delta x_1 \quad \text{(for overhang)} = \frac{ax_1^2}{24EI} \]

25. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD ON OVERHANG

\[ R_1 = V_1 = \frac{wa^2}{2l} \]

\[ R_2 = V_2 + V_3 = \frac{wa}{2l} \]

\[ V_x \quad \text{(for overhang)} = \frac{wa}{2} \]

\[ M_{x} \quad \text{(between supports)} = \frac{wa^2}{2} \]

\[ M_{x} \quad \text{(for overhang)} = \frac{wa^2}{2l} \]

\[ \Delta x \quad \text{(between supports)} = \frac{ax_1^2}{24EI} \]

\[ \Delta x_1 \quad \text{(for overhang)} = \frac{ax_1^2}{24EI} \]

26. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT END OF OVERHANG

\[ R_1 = V_1 = \frac{Pa}{l} \]

\[ R_2 = V_2 + V_3 = \frac{P}{l} \]

\[ V_x \quad \text{(between supports)} = \frac{Pa}{l} \]

\[ M_{x} \quad \text{(for overhang)} = \frac{Pa^2}{9}\sqrt{3EI} \]

\[ \Delta x \quad \text{(between supports)} = \frac{P}{l} (l + a) \]

\[ \Delta x \quad \text{(for overhang)} = \frac{Pa^2}{9}\sqrt{3EI} \]

27. BEAM OVERHANGING ONE SUPPORT—UNIFORMLY DISTRIBUTED LOAD BETWEEN SUPPORTS

\[ \text{Total Equiv. Uniform Load} = \frac{wl}{2} \]

\[ V_x = \frac{w}{2} \left( \frac{l}{2} - x \right) \]

\[ M_{\text{max.}} \quad \text{(at center)} = \frac{wl^2}{8} \]

\[ \Delta x \quad \text{(at center)} = \frac{5wl^4}{384EI} \]

\[ \Delta x_1 \quad \text{(for overhang)} = \frac{ax_1^2}{24EI} \]

28. BEAM OVERHANGING ONE SUPPORT—CONCENTRATED LOAD AT ANY POINT BETWEEN SUPPORTS

\[ \text{Total Equiv. Uniform Load} = \frac{8Pab}{l} \]

\[ R_1 = V_1 \quad \text{(max. when } a < b ) = \frac{l}{8} \]

\[ R_2 = V_2 \quad \text{(max. when } a > b ) = \frac{P}{l} \]

\[ M_{\text{max.}} \quad \text{(at point of load)} = \frac{Pab}{l} \]

\[ M_{x} \quad \text{(when } x < a) = \frac{Pbx}{2} \]

\[ \Delta x \quad \text{(at point of load)} = \frac{Pab^2}{2EI} \]

\[ \Delta x \quad \text{(when } x > a) = \frac{Pbx}{2} \]

\[ \Delta x_1 \quad \text{(for overhang)} = \frac{Pbx}{2} \]

\[ \Delta x_1 \quad \text{(for overhang)} = \frac{Pbx}{2} \]
29. CONTINUOUS BEAM—TWO EQUAL SPANS—UNIFORM LOAD ON ONE SPAN

Total Eqv. Uniform Load = \( \frac{49}{64} wL \)

\[ R_1 = V_1 = \frac{wL}{2} - \frac{M_i - M_j}{l} \]

\[ R_2 = V_2 = \frac{5}{8} wL \]

\[ R_3 = V_3 = \frac{1}{16} wL \]

\[ V_4 = \frac{9}{16} wL \]

\[ M_{\text{max.}} \text{ at } x = \frac{7}{16} L = \frac{49}{652} wL \]

\[ M_4 \text{ (at support } R_4) = \frac{1}{16} wL^2 \]

\[ M_x \text{ (when } x < l) = 16 \left( 7L - 8x \right) \]

\[ \Delta \text{ Max. (0.472 l from } R_4) = 0.0092 \frac{wL^2}{EI} \]

30. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT CENTER OF SPAN

Total Eqv. Uniform Load = \( \frac{13}{32} P \)

\[ R_1 = V_1 = \frac{13}{32} P \]

\[ R_2 = V_2 + V_3 = \frac{11}{16} P \]

\[ R_3 = V_3 = \frac{19}{32} P \]

\[ V_4 = \frac{19}{32} P \]

\[ M_{\text{max.}} \text{ at point of load} = \frac{13}{64} P L \]

\[ M_4 \text{ (at support } R_4) = \frac{3}{32} P L \]

\[ \Delta \text{ Max. (0.480 l from } R_4) = 0.015 \frac{PL^2}{EI} \]

31. CONTINUOUS BEAM—TWO EQUAL SPANS—CONCENTRATED LOAD AT ANY POINT

\[ R_1 = V_1 = \frac{PB}{4L} \left( 4L^2 - a (L + a) \right) \]

\[ R_2 = V_2 + V_3 = \frac{PB}{2L} \left( 2L^2 + b (L + a) \right) \]

\[ R_3 = V_3 = \frac{Pb}{4L} \left( L + a \right) \]

\[ V_4 = \frac{Pb}{4L} \left( 4L^2 + b (L + a) \right) \]

\[ M_{\text{max.}} \text{ at point of load} = \frac{Pb}{4L} \left( 4L^2 - a (L + a) \right) \]

\[ M_4 \text{ (at support } R_4) = \frac{Pb}{4L} \left( L + a \right) \]

32. BEAM—UNIROMLY DISTRIBUTED LOAD AND VARIABLE END MOMENTS

\[ R_1 = V_1 = \frac{wL}{2} - \frac{M_i - M_j}{l} \]

\[ R_2 = V_2 = \frac{wL}{2} - \frac{M_i - M_j}{l} \]

\[ M_1 \text{ (at } x = \frac{l}{2} + \frac{M_i - M_j}{l}) \]

\[ = \frac{wL^2}{8} - \frac{M_i + M_j}{2} + \frac{(M_i - M_j)^2}{2wL} \]

\[ M_x = \frac{wL}{2} (l - x) + \frac{M_i - M_j}{l} x - M_i \]

\[ b \text{ (location of inflection points)} = \sqrt{\frac{144}{4} \left( \frac{M_i + M_j}{w} \right) + \left( \frac{M_i - M_j}{w} \right)^2} \]

\[ \Delta = \frac{wL^2}{24EI} \left[ i^2 - \left( \frac{2l_1 + 4M_i}{wL} \right) \right] \]

33. BEAM—CONCENTRATED LOAD AT CENTER AND VARIABLE END MOMENTS

\[ R_1 = V_1 = \frac{P_1}{2} + \frac{M_i - M_j}{l} \]

\[ R_2 = V_2 = \frac{P_2}{2} - \frac{M_i - M_j}{l} \]

\[ M_1 \text{ (at center)} = \frac{P_1}{4} - \frac{M_i + M_j}{2} \]

\[ M_x \text{ (when } x < \frac{l}{2}) = \left( \frac{P_1}{2} + \frac{M_i - M_j}{l} \right) x - M_i \]

\[ M_x \text{ (when } x > \frac{l}{2}) = \frac{P_2}{2} (l - x) + \frac{(M_i - M_j)x}{l} - M_i \]

\[ \Delta \text{ (when } x < \frac{l}{2}) = \frac{P_1}{48EI} \left[ 3i^2 - 4x_1^2 - 8l (l - x) \right] \left( \frac{M_i (2l - x) + M_j (l + x)}{l} \right) \]