Examples:
Plate and Grids

Example 1

What is the maximum positive and negative bending moments developed in a 52 x 40 ft fully fixed plate that carries a load of 120 lb/ft^2?

SOLUTION:
The aspect ratio of the side lengths, a/b, must be determined and an appropriate coefficient chart must be found:

a/b = 52/40 = 1.3 (no units, and a is always the bigger number).

The coefficients for moment for the a side length and b side length for fixed support all sides and a/b = 1.3 are:

C_a = +0.0131 and C_a = -0.0333
C_b = +0.0327 and C_b = -0.0687

The maximum moments are calculated with the formula in the table:

\[ M_a(\text{positive}) = C_a w a^2 = 0.0131(120 \frac{lb}{ft})(52 \text{ ft})^2 = 4251 \frac{lb\cdot ft^2}{ft} \]

\[ M_a(\text{negative}) = C_a w a^2 = -0.0333(120 \frac{lb}{ft})(52 \text{ ft})^2 = -10,805 \frac{lb\cdot ft^2}{ft} \]

\[ M_b(\text{positive}) = C_b w b^2 = 0.0327(120 \frac{lb}{ft})(40 \text{ ft})^2 = 6278 \frac{lb\cdot ft^2}{ft} \]

\[ M_b(\text{negative}) = C_b w b^2 = -0.0687(120 \frac{lb}{ft})(40 \text{ ft})^2 = -13,190 \frac{lb\cdot ft^2}{ft} \]
Example 2
A two-way interior-bay flat (concrete) slab with the dimensions shown supports a live loading of 80 lb/ft² and has a dead load of 90 lb/ft². The columns can be assumed to be 18 inches square. Determine the design moments based on ACI-318, (ASCE-7) and the Direct Design method.

Also compare design moments for an exterior-interior bay

SOLUTION:
Determine the distributed load combinations:
\[ w_u = 1.2D + 1.6L = 1.2(90 \text{ lb/ft}^2) + 1.6(80 \text{ lb/ft}^2) = 236 \text{ lb/ft}^2 \]

Determine the clear span length for the N-S direction:
\[ l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} \]
\[ = 25 \text{ ft} - \frac{1}{2} (18 \text{ in/12 in/ft}) - \frac{1}{2} (18 \text{ in/12 in/ft}) = 23.5 \text{ ft} \]

Because \( l_2 \) is not the same width on either side of an interior panel, it is taken as the average = \((21 \text{ ft} + 20 \text{ ft})/2 = 20.5 \text{ ft} \).

Total moment (to distribute to middle and interior column strip):
\[ M_o = w_u l_2 l_n^2 / 8 = \left( \frac{236 \text{ lb/ft}^2 \cdot 20.5 \text{ ft} \cdot 23.5 \text{ ft}}{8} \right) = 333,973 \text{ lb-ft}^2 \]

Interior Column Strip \( l_2 \leq l_1 \):

The column strip width is \( \frac{1}{4} \) the smaller of \( l_2 \) either side of the column:

strip width = \( \frac{1}{4} \) (21 ft) + \( \frac{1}{4} \) (20 ft) = 10.25 ft

From Table 4.2, the maximum positive moment occurs in an end span:
\[ M( \text{ positive } ) = 0.31 M_o = (0.31)(333,973 \text{ lb-ft}^2) = 103,532 \text{ lb-ft/(10.25 ft)} \]
\[ = 10,101 \text{ lb-ft/ft} \]

The positive design moment for an interior span is:
\[ M( \text{ positive } ) = 0.21 M_o = (0.21)(333,973 \text{ lb-ft}^2) = 70,134 \text{ lb-ft/(10.25 ft)} \]
\[ = 6842 \text{ lb-ft/ft} \]

Table 4-2 Flat Plate or Flat Slab Supported Directly on Columns

<table>
<thead>
<tr>
<th>Slab Moments</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>End Span</td>
<td>Interior Span</td>
<td>End Span</td>
<td>Interior Span</td>
<td>End Span</td>
</tr>
<tr>
<td>Total Moment</td>
<td>0.26 M_o</td>
<td>0.52 M_o</td>
<td>0.70 M_o</td>
<td>0.35 M_o</td>
<td>0.65 M_o</td>
</tr>
<tr>
<td>Column Strip</td>
<td>0.26 M_o</td>
<td>0.31 M_o</td>
<td>0.53 M_o</td>
<td>0.21 M_o</td>
<td>0.49 M_o</td>
</tr>
<tr>
<td>Middle Strip</td>
<td>0.21 M_o</td>
<td>0.17 M_o</td>
<td>0.14 M_o</td>
<td>0.16 M_o</td>
<td></td>
</tr>
</tbody>
</table>

Note: All negative moments are at face of support.
From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

\[ M(\text{negative}) = 0.53M_o = (0.53)(333,973^{lb-ft^2}) = 177,006^{lb-ft^2}, \text{ distributed over 10.25 ft = 177,006 lb-ft/(10.25 ft)} = 17,269 \text{ lb-ft/ft} \]

The negative design moment at the exterior of an end span is:

\[ M(\text{negative}) = 0.26M_o = (0.26)(333,973^{lb-ft^2}) = 86,833^{lb-ft^2}, \text{ distributed over 10.25 ft = 86,833 lb-ft/(10.25 ft)} = 8472 \text{ lb-ft/ft} \]

The negative design moment for an interior span is:

\[ M(\text{negative}) = 0.49M_o = (0.49)(333,973^{lb-ft^2}) = 163,647^{lb-ft^2}, \text{ distributed over 10.25 ft = 163,647 lb-ft/(10.25 ft)} = 15,966 \text{ lb-ft/ft} \]

\[ \text{Middle Strip:} \]

The width is the remaining width of \( l_z \) between column strips:

\[ \text{strip width} = 21 \text{ ft} - \frac{1}{4} (20 \text{ ft}) - \frac{1}{4} (21 \text{ ft}) = 10.75 \text{ ft} \]

From Table 4.2, the maximum positive moment occurs in an end span:

\[ M(\text{positive}) = 0.21M_o = (0.21)(333,973^{lb-ft^2}) = 70,134^{lb-ft^2}, \text{ distributed over 10.75 ft = 70,134 lb-ft/(10.75 ft)} = 6524 \text{ lb-ft/ft} \]

The positive design moment for an interior span is:

\[ M(\text{positive}) = 0.14M_o = (0.14)(333,973^{lb-ft^2}) = 46,756^{lb-ft^2}, \text{ distributed over 10.75 ft = 46,756 lb-ft/(10.75 ft)} = 4349 \text{ lb-ft/ft} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

\[ M(\text{negative}) = 0.17M_o = (0.17)(333,973^{lb-ft^2}) = 56,775^{lb-ft^2}, \text{ distributed over 10.75 ft = 56,775 lb-ft/(10.75 ft)} = 5281 \text{ lb-ft/ft} \]

There is no negative design moment at the exterior of an end span.

The negative design moment for an interior span is:

\[ M(\text{negative}) = 0.16M_o = (0.16)(333,973^{lb-ft^2}) = 53,436^{lb-ft^2}, \text{ distributed over 10.75 ft = 53,436 lb-ft/(10.75 ft)} = 4971 \text{ lb-ft/ft} \]

\[ \text{Exterior Column Strip:} \]

The value to use for \( l_z \) for an edge strip includes the distance to the outside of the columns = 21 ft + \( \frac{1}{2} \) (18 in/12 in/ft) = 21.75 ft

\[ M_o = \frac{w_{w}l_z^2}{8} = \frac{(236^{lb-ft^2})(21.75 \text{ ft})(23.5 \text{ ft})^2}{8} = 354,337^{lb-ft^2} \]

The width is \( \frac{1}{4} \) \( l_z \) one side of the column plus the distance to the slab edge:

\[ \text{strip width} = \frac{1}{4} \text{ (21 ft)} + \frac{1}{2} \text{ (18 in/12 in/ft)} = 6 \text{ ft} \]

So a comparison to the interior column strip maximum positive moment occurring in an end span is:

\[ M(\text{positive}) = 0.31M_o = (0.31)(354,337^{lb-ft^2}) = 109,844^{lb-ft^2}, \text{ distributed over 6 ft = 109,844 lb-ft/(6 ft)} = 18,307 \text{ lb-ft/ft} \]

(as opposed to 10,101 lb-ft/ft)
For the E-W direction:

Because the adjacent spans are not the same length, the longer span, which is the END span will be larger:

\[ l_n = l_1 - \frac{1}{2} \text{ column width} - \frac{1}{2} \text{ column width} \]
\[ = 21 \text{ ft} - \frac{1}{2} (18 \text{ in/12 in/ft}) - \frac{1}{2} (18 \text{ in/12 in/ft}) = 19.5 \text{ ft} \]

Because \( l_2 \) is 25 ft.

Total moment (to distribute to middle and interior column strip):

\[ M_o = \frac{w_o l_2 l_n^2}{8} = \frac{(23.6 \text{ lb/ft})(25 \text{ ft})(19.5 \text{ ft})^2}{8} = 280,434 \text{ lb-ft} \]

Interior Column Strip END Spans (\( l_2 > l_1 \)):

The column strip width is \( \frac{1}{4} \) the smaller of \( l_1 \) and \( l_2 \) either side of the column:

strip width = \( \frac{1}{4} \) (21 ft) + \( \frac{1}{4} \) (21 ft) = 10.5 ft

From Table 4.2, the maximum positive moment occurs in an end span:

\[ M(\text{ positive }) = 0.31M_o = (0.31)(280,434 \text{ lb-ft}) = 86,935 \text{ lb-ft/(10.5 ft)} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

\[ M(\text{ negative }) = 0.53M_o = (0.53)(280,434 \text{ lb-ft}) = 148,630 \text{ lb-ft/(10.5 ft)} \]

The negative design moment at the exterior of an end span is:

\[ M(\text{ negative }) = 0.26M_o = (0.26)(280,434 \text{ lb-ft}) = 72,913 \text{ lb-ft/(10.5 ft)} \]

Middle Strip END Spans:

The width is the remaining width of \( l_2 \) between column strips:

strip width = 25 ft - \( \frac{1}{4} \) (21 ft) - \( \frac{1}{4} \) (21 ft) = 14.5 ft

From Table 4.2, the maximum positive moment occurs in an end span:

\[ M(\text{ positive }) = 0.21M_o = (0.21)(280,434 \text{ lb-ft}) = 58,891 \text{ lb-ft/(14.5 ft)} \]

From Table 4.2, the maximum negative moment occurs in an end span at the first interior column face:

\[ M(\text{ negative }) = 0.17M_o = (0.17)(280,434 \text{ lb-ft}) = 47,674 \text{ lb-ft/(14.5 ft)} \]

There is no negative design moment at the exterior of an end span.
Exterior Column Strip END Spans:

The value to use for $l_2$ for an edge strip includes the distance to the outside of the columns = 25 ft + $\frac{1}{2}$ (18 in/12 in/ft) = 25.75 ft

$$M_o = \frac{w_o l_2^2}{8} = \left( \frac{236 \text{ lb/ft} \times 25.75 \text{ ft} \times 19.5 \text{ ft}}{8} \right) = 288,847 \text{ lb-ft}$$

The width is $\frac{3}{4} l_1$ (because it is smaller than $l_2$) one side of the column plus the distance to the slab edge:

strip width = $\frac{3}{4} (21 \text{ ft}) + \frac{1}{2} (18 \text{ in/12 in/ft}) = 6 \text{ ft}$

So a comparison to the interior column END strip maximum positive moment occurring in an end span is:

$$M(\text{ positive}) = 0.31 M_o = (0.31)(288,847 \text{ lb-ft}) = 89,543 \text{ lb-ft/ft}$$

distributed over 6 ft = $\frac{89,543 \text{ lb-ft}}{6 \text{ ft}} = 14,923 \text{ lb-ft/ft}$

(as opposed to 8279 lb-ft/ft)

### TABLE OF DESIGN MOMENTS

<table>
<thead>
<tr>
<th>slab moments / ft</th>
<th>End Span</th>
<th>Interior Span</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Exterior Negative</td>
<td>Positive</td>
</tr>
<tr>
<td><strong>NS column strip - interior</strong></td>
<td>8472 lb-ft/ft</td>
<td>10,101 lb-ft/ft</td>
</tr>
<tr>
<td><strong>NS middle strip</strong></td>
<td>0</td>
<td>6524 lb-ft/ft</td>
</tr>
<tr>
<td><strong>NS column strip - edge</strong></td>
<td>15,355 lb-ft/ft</td>
<td>18,307 lb-ft/ft</td>
</tr>
<tr>
<td><strong>EW column strip - interior</strong></td>
<td>6944 lb-ft/ft</td>
<td>8279 lb-ft/ft</td>
</tr>
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<td><strong>EW middle strip</strong></td>
<td>0</td>
<td>4061 lb-ft/ft</td>
</tr>
<tr>
<td><strong>EW column strip - edge</strong></td>
<td>12,517 lb-ft/ft</td>
<td>14,923 lb-ft/ft</td>
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