**reinforced concrete construction**

**Concrete Construction**
- cast-in-place
- tilt-up
- prestressing
- post-tensioning

**Concrete Materials**
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
Concrete Materials

- reinforcement
  - deformed bars
  - prestressing strand
  - stirrups
  - development length
  - anchorage
  - splices

Concrete Beams

- types
  - reinforced
  - precast
  - prestressed

- shapes
  - rectangular, I
  - T, double T's, bulb T's
  - box
  - spandrel

Concrete Materials

- fire resistance
  - most fire-resistant structural material
  - low rate of penetration
  - retains strength if exposure not too long
    - stable to 900 – 1200 °F internally
    - loses 50% after that
  - no toxic fumes
  - cover necessary to protect steel

Concrete Beams

- deformation
  - camber (elastic)
    - hogging ↑
    - sagging ↓
  - shrinkage strain
    - 200-400 x 10^-6
    - about 2-3 years
  - creep strain
    - 2~3 times elastic strain
    - about 2-3 years
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°
- bearing
  - crushing

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for failure
  - strength design (LRFD)
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = \( f'c \)

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

Transformation of Material

- \( n \) is the ratio of \( E \)'s
  \[ n = \frac{E_2}{E_1} \]
- effectively widens a material to get same stress distribution

\[
f_1 = E_1 \varepsilon = -\frac{E_1 y}{R} \quad f_2 = E_2 \varepsilon = -\frac{E_2 y}{R}
\]
Stresses in Composite Section

- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by \( n \)

\[
\begin{align*}
  n &= \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}} \\
  f_c &= -\frac{M_y}{I_{\text{transformed}}} \\
  f_s &= -\frac{M_{yn}}{I_{\text{transformed}}}
\end{align*}
\]

Reinforced Concrete Analysis

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

Reinforced Concrete - stress/strain

Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for \( x \)

\[
 bx \cdot \frac{x}{2} - nA_s (d - x) = 0
\]
**T sections**

- n.a. equation is different if n.a. below flange

\[
b_f h_f \left( x - \frac{h_f}{2} \right) + \left( x - h_f \right) b_w \frac{x - h_f}{2} - nA_s (d - x) = 0
\]

**ACI Load Combinations**

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

**Reinforcement**

- deformed steel bars (rebar)
  - Grade 40, $F_y = 40$ ksi
  - Grade 60, $F_y = 60$ ksi - most common
  - Grade 75, $F_y = 75$ ksi
  - US customary in # of 1/8” φ
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

**Reinforced Concrete Design**

- stress distribution in bending

\[ A_x \]

Wang & Salmon, Chapter 3
Force Equations

- \( C = 0.85 \, f'_c \, b \, a \)
- \( T = A_s \, f_y \)

where
- \( f'_c \) = concrete compressive strength
- \( a \) = height of stress block
- \( \beta_1 \) = factor based on \( f'_c \)
- \( c \) = location to the n.a.
- \( b \) = width of stress block
- \( f_y \) = steel yield strength
- \( A_s \) = area of steel reinforcement

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
- \( d \) = depth to the steel n.a.
- \( A_s \)

with \( \phi = 0.65 + (\varepsilon_y - \varepsilon_s) \frac{0.25}{(0.005 - \varepsilon_s)} \geq 0.65 \)

- \( M_u \leq \phi M_n \)
- \( \phi = 0.9 \) for flexure

\( \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)

Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield
- reinforcement ratio
  - \( \rho = \frac{A_s}{b d} \)
  - use as a design estimate to find \( A_s, b, d \)
  - max \( \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))
  - *with \( \varepsilon_{\text{steel}} \geq 0.005, \phi = 0.9 \)

\( A_s \) for a given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 \, f'_c \, b \, a}{f_y} \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y (d-a/2) \)
    \[ a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
  4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
**A_s For Given Section (cont)**

- chart method
  - Wang & Salmon
  - Fig. 3.8.1  \( R_n \) vs. \( \rho \)
  1. calculate \( R_n = \frac{M_n}{bd^2} \)
  2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
  3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

**Shear in Concrete Beams**

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

**ACI Shear Values**

- \( V_u \) is at distance \( d \) from face of support
- shear capacity: \( V_c = \nu c \times b_w d \)
  - where \( b_w \) means thickness of web at n.a.
- shear stress (beams)
  - \( \nu c = 2\lambda \sqrt{f'_c} \)
  - \( \phi = 0.75 \) for shear
  - \( \phi V_c = \phi 2\lambda \sqrt{f'_c} b_w d \)
    - \( f'_c \) is in psi
    - \( \lambda \) for lightweight materials
- shear strength: \( V_u \leq \phi V_c + \phi V_s \)
  - \( V_s \) is strength from stirrup reinforcement

**Stirrup Reinforcement**

- shear capacity:
  \[
  V_s = \frac{A_v f_y d}{s} \leq 8 f'_c b_w d
  \]
  - \( A_v \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup
- may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>Stirrup spacing, d</th>
<th>Required</th>
<th>Recommended MinimumI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>--</td>
</tr>
<tr>
<td>Max (ACI 9.7 R ??)</td>
<td>--</td>
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</tr>
<tr>
<td></td>
<td>d/2 or 24 in.</td>
<td>d/2 or 24 in. for (V_u - V_c) ≤ 0.5 f'_c b_w d</td>
</tr>
<tr>
<td></td>
<td>d/4 or 12 in. for (V_u - V_c) &gt; 0.5 f'_c b_w d</td>
<td>d/4 or 12 in. for (V_u - V_c) &gt; 0.5 f'_c b_w d</td>
</tr>
</tbody>
</table>

*Members subjected to shear in flexure only: V_u = (2d/UI b_w d + 0.75 (ACI 21.5.5.1))
**A_L = 2 A_y for L/8 stirups: f_u ≤ 60 ksf (ACI 20.2.2.4)
I Maximum spacing based on minimum shear reinforcement (A_L, c/50 f'_c or A_L, c/0.75 (f_u))

Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete (~ 145 lb/ft$^3$)
      $$ E_c = 57,000 \sqrt{f'_c} $$
    - concrete between 90 and 155 lb/ft$^3$
      $$ E_c = w c^{1.5} 33 \sqrt{f'_c} $$
- cracked
  - I cracked
  - $E$ adjusted

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these
  - L/180 roof systems (typical) – live
  - L/240 floor systems (typical) – live + long term
  - L/360 supporting plaster – live
  - L/480 supporting masonry – live + long term

Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit
Prestressed Concrete

- pretensioned
  - reinforcement bonded
- post-tensioned
  - bonded or unbonded
  - end bearing
- precast
  - concrete premade in a position other than its final position in the structure

Prestressed Concrete

- high strength tendons
  - grade 250
  - grade 270

\[
g_{tt} \frac{Mc}{A} P_{f} I = g_{b} \frac{Mc}{A} P_{f} I + \frac{P}{A} \frac{Mc}{I_g} + \frac{I_g}{I} \frac{Mc}{I_g} P_{e} c_{b} + \frac{I_g}{I} \frac{Mc}{I_g} P_{e} c_{b} + \frac{I_g}{I} \frac{Mc}{I_g} P_{e} c_{b}
\]
**Prestressed Concrete**

![Diagram of prestressed concrete](image)

*Figure 4.2* Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full dead load plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

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**Composite Beams**

- concrete
  - in compression
- steel
  - in tension
- shear studs

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**Continuous Beams**

- reduced size
- reduced moments
- moments can reverse with loading patterns
- need top & bottom reinforcement
- sensitive to settlement
Concrete Columns

- columns require
  - ties or spiral reinforcement to “confine” concrete (#3 bars minimum)
  - minimum amount of longitudinal steel (4 bars minimum)

Concrete Columns

- effective length in monolithic casts must be found with respect to stiffness of joint
- not slender when
  \[ \frac{kL}{r} < 22 \]
  *not braced
Concrete Columns

- $P_o$ – no bending
  \[ P_o = 0.85 f'_c(A_g - A_{st}) + f_y A_{st} \]
- $\phi_c = 0.65$ for ties with $P_n = 0.8P_o$
- $\phi_c = 0.70$ for spirals with $P_n = 0.85P_o$
- $P_u \leq \phi_c P_n$
- nominal axial capacity:
  - presumes steel yields
  - concrete at ultimate stress

Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection ($P - \Delta$)

Columns with Bending

- for ultimate strength behavior, ultimate strains can’t be exceeded
  - concrete 0.003
  - steel $\frac{f_y}{E_s}$
- $P$ reduces with $M$

Columns with Bending

- need to consider combined stresses
  \[ \frac{P_n}{P_o} + \frac{M_n}{M_o} \leq 1 \]
- plot interaction diagram

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*Figure 10.6: Considerations for development of bending in steel columns: (a) bending induced by eccentric load, (b) bending transferred to column in a rigid frame, and (c) combined loading condition, separately producing axial compression and bending.*

*Figure 13.6.1: Typical strength interaction diagram for axial compression and bending moment about one axis. Transition zone is when $\varepsilon_s < \varepsilon_c < 0.003$.*

*Figure 5-3: Transition Stages on Interaction Diagram*
Concrete Floor Systems

- types & spanning direction

- one-way and two-way moments
- flexure design as T-beams (+/ M)
- increase of 10% $V_c$ permitted
- slabs need steel
- effective width is
  - $L/4$
  - $b_w + 16t$
  - center-to-center of beams

One-way Joists
- standard stems
- 2.5” to 4.5” slab
- ~30” widths
- reusable forms
One-way Joists

- wide pans
- 5', 6' up
- light loads & long spans
- one-leg stirrups

Two-way Joists

- domed pans
- 3', 4' & 5'

Construction Supervision

- proper placement of all reinforcement
  - welding
  - splices
- mix design
  - slump
- in-situ strength
  - cast cylinders
  - cylinder cores – if needed