reinforced concrete construction

Concrete Construction
- cast-in-place
- tilt-up
- prestressing
- post-tensioning

Concrete Materials
- low strength to weight ratio
- relatively inexpensive
  - Portland cement
  - aggregate
  - water
Concrete Materials

• reinforcement
  – deformed bars
  – prestressing strand
  – stirrups
  – development length
  – anchorage
  – splices

Concrete Beams

• types
  – reinforced
  – precast
  – prestressed

• shapes
  – rectangular, I
  – T, double T’s, bulb T’s
  – box
  – spandrel

Concrete Materials

• fire resistance
  – most fire-resistive structural material
  – low rate of penetration
  – retains strength if exposure not too long
    • stable to 900 – 1200 °F internally
    • loses 50% after that
  – no toxic fumes
  – cover necessary to protect steel

Concrete Beams

• deformation
  – camber (elastic)
    • hogging ↑
    • sagging ↓
  – shrinkage strain
    • 200-400 x 10^-6
    • about 2-3 years
  – creep strain
    • 2~3 times elastic strain
    • about 2-3 years
Concrete Beams

- shear
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°

- bearing
  - crushing

Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for failure
  - strength design (LRFD)
    - service loads x load factors
    - concrete holds no tension
    - failure criteria is yield of reinforcement
    - failure capacity x reduction factor
    - factored loads < reduced capacity
  - concrete strength = $f'_c$

Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

Transformation of Material

- $n$ is the ratio of $E$'s
- $n = \frac{E_2}{E_1}$
- effectively widens a material to get same stress distribution

\[
f_1 = E_1 \varepsilon = -\frac{E_1 y}{R} \quad f_2 = E_2 \varepsilon = -\frac{E_2 y}{R}
\]
**Stresses in Composite Section**

- with a section transformed to one material, new I
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by n

\[
n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}
\]

\[
f_c = -\frac{M_y}{I_{\text{transformed}}}
\]

\[
f_s = -\frac{M_{yn}}{I_{\text{transformed}}}
\]

**Reinforced Concrete Analysis**

- for stress calculations
  - steel is transformed to concrete
  - concrete is in compression above n.a. and represented by an equivalent stress block
  - concrete takes no tension
  - steel takes tension
  - force ductile failure

**Reinforced Concrete - stress/strain**

Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.

**Location of n.a.**

- ignore concrete below n.a.
- transform steel
- same area moments, solve for x

\[
bx \cdot \frac{x}{2} - nA_s (d - x) = 0
\]
T sections

- n.a. equation is different if n.a. below flange

\[
b_f h_f \left( x - \frac{h_f}{2} \right) + \left( x - h_f \right) b_w \left( x - h_f \right) \frac{2}{2} - nA_x (d - x) = 0
\]

Reinforcement

- deformed steel bars (rebar)
  - Grade 40, \( F_y = 40 \text{ ksi} \)
  - Grade 60, \( F_y = 60 \text{ ksi} \) - most common
  - Grade 75, \( F_y = 75 \text{ ksi} \)
  - US customary in # of 1/8” \( \phi \)
- longitudinally placed
  - bottom
  - top for compression reinforcement
  - spliced, hooked, terminated...

ACI Load Combinations*

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors

Reinforced Concrete Design

- stress distribution in bending

\[
\begin{align*}
A_s & \ldots \\
A_c & \ldots \\
\text{actual stress} & \ldots \\
\text{Whitney stress} & \ldots \\
\end{align*}
\]

Wang & Salmon, Chapter 3
Force Equations

- \( C = 0.85 \beta \frac{f'}{c} ba \)
- \( T = A_s f_y \)

where

- \( f' \) = concrete compressive strength
- \( a \) = height of stress block
- \( \beta \) = factor based on \( f' \)
- \( c \) = location to the n.a.
- \( b \) = width of stress block
- \( f_y \) = steel yield strength
- \( A_s \) = area of steel reinforcement

Over and Under-reinforcement

- over-reinforced
  - steel won’t yield
- under-reinforced
  - steel will yield

reinforcement ratio

- \( \rho = \frac{A_s}{bd} \)
- use as a design estimate to find \( A_s, b, d \)
- max \( \rho \) is found with \( \epsilon_{steel} \geq 0.004 \) (not \( \rho_{bal} \))
- *with \( \epsilon_{steel} \geq 0.005, \phi = 0.9 \)

Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d \) = depth to the steel n.a.
- with \( A_s \)

  - \( a = A_s f_y \)
  - \( \phi \) = 0.65 + \( \epsilon_{steel} - \epsilon_{y} \)
  - \( \phi \) = 0.9 for flexure*
  - \( \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)

\( A_s \) for a given Section

- several methods
  - guess \( a \) and iterate
    1. guess \( a \) (less than n.a.)
    2. \( A_s = \frac{0.85 f' c b a}{f_y} \)
        \( A_s \)
    3. solve for \( a \) from \( M_u = \phi A_s f_y (d-a/2) \)
        \( a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \)
    4. repeat from 2. until \( a \) from 3. matches \( a \) in 2.
A_s For Given Section (cont)

- chart method
  - Wang & Salmon
    Fig. 3.8.1  \( R_n vs. \rho \)
  1. calculate \( R_n = \frac{M_n}{bd^2} \)
  2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)
  3. calculate \( A_s \) and \( a \)
- simplify by setting \( h = 1.1d \)

Shear in Concrete Beams

- flexure combines with shear to form diagonal cracks
- horizontal reinforcement doesn’t help
- stirrups = vertical reinforcement

ACI Shear Values

- \( V_u \) is at distance \( d \) from face of support
- shear capacity: \( V_c = \nu_c \times b_w d \)
  - where \( b_w \) means thickness of web at n.a.
- shear stress (beams)
  - \( \nu_c = 2\lambda \sqrt{f'_c} \)
  - \( \phi = 0.75 \) for shear
  - \( f'_c \) is in psi
  - \( \nu V_c = \phi 2\lambda \sqrt{f'_c} b_w d \)
  - \( \lambda \) for lightweight materials
- shear strength: \( V_u \leq \phi V_c + \phi V_s \)
  - \( V_s \) is strength from stirrup reinforcement

Stirrup Reinforcement

- shear capacity:
  \[ V_s = \frac{A_V f_y d}{s} \leq 8 \sqrt{f'_c b_w d} \]
  - \( A_V \) = area in all legs of stirrups
  - \( s \) = spacing of stirrup
- may need stirrups when concrete has enough strength!
Required Stirrup Reinforcement

- spacing limits

<table>
<thead>
<tr>
<th>Table 3-8 ACI Provisions for Shear Design*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_c &lt; \frac{V_{u,2}}{2}$</td>
</tr>
<tr>
<td>Required area of stirrup, $A_{sb}$**</td>
</tr>
<tr>
<td>Required</td>
</tr>
<tr>
<td>Recommended **</td>
</tr>
<tr>
<td>Minimum†</td>
</tr>
<tr>
<td>Maximum†</td>
</tr>
</tbody>
</table>

*Members subjected to shear and flexure only: $V_u = 2\sqrt{f'_c} b_w d + 0.75$ (ACI 9.7.8.2.4)
**$A_{sb} = 2 \times A_{sb}$ for I stirrup, $f_s = 60$ ksi (ACI 20.2.2.2)
†A practical limit for minimum spacing is $d_f$
‡Maximum spacing based on minimum shear reinforcement $f' = A_{sb} / 60b_w$ or $A_{sb} / 0.75b_w$

Concrete Deflections

- elastic range
  - I transformed
  - $E_c$ (with $f'_c$ in psi)
    - normal weight concrete ($\sim 145$ lb/ft$^3$)
      $$E_c = 57,000\sqrt{f'_c}$$
    - concrete between 90 and 155 lb/ft$^3$
      $$E_c = w_c^{1.5} 33\sqrt{f'_c}$$
- cracked
  - I cracked
  - $E$ adjusted

Deflection Limits

- relate to whether or not beam supports or is attached to a damageable non-structural element
- need to check service live load and long term deflection against these

| $L/180$ | roof systems (typical) – live |
| $L/240$ | floor systems (typical) – live + long term |
| $L/360$ | supporting plaster – live |
| $L/480$ | supporting masonry – live + long term |

Prestressed Concrete

- impose a longitudinal force on a member in order to withstand more loading until the member reaches a tensile limit
Prestressed Concrete

- **pretensioned**
  - reinforcement bonded

- **post-tensioned**
  - bonded or unbonded
  - end bearing

- **precast**
  - concrete premade in a position other than its final position in the structure

Prestressed Concrete

- **high strength tendons**
  - grade 250
  - grade 270

\[
M(\omega) = \begin{cases} 
P & \text{top} \\
\frac{P}{A} I_g & \text{bottom} \\
\frac{P}{A} I_g + I_g & \text{at fiber}
\end{cases}
\]

\[
f' = \frac{P}{A} \frac{P e c_b}{I_g} \frac{M c_b}{I_g} = \frac{P}{A} \left(1 + \frac{e c_b}{r} \right) \frac{M c_b}{I_g}
\]

\[
f_b = \frac{P}{A} \frac{P e c_b}{I_g} \frac{M c_b}{I_g} + \frac{P}{A} \left(1 + \frac{e c_b}{r} \right) \frac{M c_b}{I_g}
\]

\[
\text{remember: } r = \frac{T}{M c_b}
\]
Prestressed Concrete

![Diagram of prestressed concrete](image1)

**Figure 4.2** Flexural stress distribution throughout loading history. (a) Beam section. (b) Initial prestressing stage. (c) Self-weight and effective prestress. (d) Full deadload plus effective prestress. (e) Full service load plus effective prestress. (f) Limit state of stress at ultimate load for underreinforced beam.

**Composite Beams**
- concrete
  - in compression
- steel
  - in tension
- shear studs

**Continuous Beams**
- reduced size
- reduced moments
- moments can reverse with loading patterns
- need top & bottom reinforcement
- sensitive to settlement
Concrete Columns

- columns require
  - ties or spiral reinforcement to “confine” concrete
    (#3 bars minimum)
  - minimum amount of longitudinal steel
    (4 bars minimum)

- effective length in monolithic casts must be found with respect to stiffness of joint

- not slender when
  \[
  \frac{kL}{r} < 22
  \]
  *not braced
Concrete Columns

- $P_o$ – no bending
  $$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st}$$

- $\phi_c = 0.65$ for ties with $P_n = 0.8P_o$
- $\phi_c = 0.70$ for spirals with $P_n = 0.85P_o$
- $P_u \leq \phi_c P_n$
- nominal axial capacity:
  - presumes steel yields
  - concrete at ultimate stress

Columns with Bending

- eccentric loads can cause moments
- moments can change shape and induce more deflection ($P - \Delta$)

$$P - \Delta$$

Columns with Bending

- for ultimate strength behavior, ultimate strains can't be exceeded
  - concrete 0.003
  - steel $\frac{f_y}{E_s}$
- $P$ reduces with $M$

$$\frac{P_n}{P_o} + \frac{M_n}{M_o} \leq 1$$

- plot interaction diagram

Figure 10.6 Considerations for development of bending in steel columns: (a) bending induced by eccentric load, (b) bending transferred to column in a rigid frame, and (c) combined loading condition, separately producing axial compression and bending.
Concrete Floor Systems

- types & spanning direction

- one-way and two-way moments
- flexure design as T-beams (+/- M)
- increase of 10% $V_c$ permitted
- slabs need steel
- effective width is
  - $L/4$
  - $b_w + 16t$
  - center-to-center of beams

One-way Joists

- standard stems
- 2.5” to 4.5” slab
- ~30” widths
- reusable forms
One-way Joists
- wide pans
- 5’, 6’ up
- light loads & long spans
- one-leg stirrups

Two-way Joists
- domed pans
- 3’, 4’ & 5’

Construction Supervision
- proper placement of all reinforcement
  - welding
  - splices
- mix design
  - slump
- in-situ strength
  - cast cylinders
  - cylinder cores – if needed