ARCH 631. Topic 5 Reading Notes

(includes Appendix 13)

- A truss is defined as an assemblage of linear elements arranged in a triangle or combination of triangles which forms a rigid member joined at the member ends with pinned connections with loads occurring at the joints
- Bending is not present with loading at joints; members are in tension or in compression axially (and what makes it efficient)
- Senses of forces (T or C) can be determined by visualizing the deformation were members to be removed; the motion indicates tension when "stretching" the length or compression when shortening it
- Zero forces members are ones with no tension or compression, but are required for stability
- An arch-and-cable analogy may also be used to visualize the force senses
- Truss analysis steps:
  determine if the geometry is stable; can have cross members (indeterminate); $n = 3+2(j-3)$
  determine member forces; equilibrium of joint or section – internal forces are "exposed"
- Equilibrium of joints requires sum of forces in x and in y to equal 0, so only two unknowns can be solved for (common to start at support point with known forces...)
- Arrows away indicate tension; arrows "into" point or member indicate compression (must assume direction when unknown with positive answer indicating assumption was validated)
- Solving two equations simultaneously is very likely in method of joints
- Zero-force members can be found by inspection, as can equal forces at a joint
- Graphic statics for trusses is based on the tip-to-tail graphical technique for vector addition (drawing to angle and scale representing the force magnitude); result is a Maxwell diagram
- Method of sections requires the section to be in equilibrium (forces AND moments) and that the geometry remain stable; cut members get internal forces "exposed"; section cut does not have to be straight
- NO MORE THAN THREE members in a section can get cut with three equations of equilibrium
- Method of sections useful to solve for one unknown force when summing moments at the intersection of the other two unknown forces
- For parallel chord trusses, the web forces can be compared to the shear of a horizontal beam while the top and bottom chord forces (with the perpendicular distance like a moment couple) can be compared to the bending moment
- Statically indeterminate trusses have too many unknown for the number of equilibrium equations that can be used; ones with one degree of indeterminacy (one more unknown than equations) can be solved by assuming a member is a cable and cannot be in compression – when it solves in tension, the assumption has been validated (when members "cross")
- Space trusses can use the tetrahedron geometry or pyramid (triangulated); all six equations of equilibrium must be satisfied

- Rigid joints can be made in trusses, but that means bending moments will develop and should be analyzed in the final design
- Analytical computer methods (like Multiframe) make the solution of truss member forces much easier; design involves making an assumption on the member sizes, analyzing and revising the sizes to be efficient
- Design criteria for trusses that determine overall shape and member sizing can include structural efficiency (which seeks to minimize the total amount of material) or construction efficiency (same members or connections)
- Comparing the bending moment and shear of a beam to the web forces and chord forces show that it is desirable to design for tension in the long web members and increase the depth to lower the maximum chord force
- Cables can be used for tensile members with care – if they go into compression, the stability can be compromised especially under varying load conditions
- Funicularly shaped trusses are shaped similarly to arches and cables in order to get a nearly constant compression or tension in the top and bottom members; lenticular trusses are efficient in that the diagonal members carry very little load, but must be crossed to be stable
- To estimate a depth based on minimizing the total volume (weight) of a truss, one rule of thumb for light loads is L/20, with secondary trusses at L/10, trusses with huge loads at L/4 or L/5
- Truss design issues:
  - loading conditions possible (whole spectrum); worst case for *each* member
  - choice of material and cross section required for tension (P/A) or for compression members (buckling stress); objective to minimize long compression members
  - may want to design for largest cross section and make the rest of the members the same
  - bracing of compression members (like top chord) is significant for shortening effective length and reducing member size
  - avoid lateral buckling of truss compression members with bracing transversely or stiffer (larger) members (I_y); or even 3D geometry
- Three dimensional truss configurations can be more efficient in two-way systems; can resist lateral buckling; can also resist torsional effects
- Columns need not be vertical, but are defined as rigid linear elements with loads applied solely at the member ends (with no bending)
- Short columns fail due to crushing while long columns fail due to buckling (instability) at low stresses
- Buckling is associated with low stiffness related to length, modulus of elasticity and the cross section
- An unbraced member will buckle about the weak axis which is that associated with the lesser ability to resist buckling (commonly I_y)
- End conditions (restraints) and bracing will change the stiffness
- Compressive stress is defined as P/A; P limit can be determined for allowable stress as A time crushing strength
- Eccentric loads are those not applied through the centroid of the cross section and result in non-linear stresses because of the addition of bending stress; $f = f_a + f_b = \pm P/A \pm (Pe)c/I$

- The limit of the eccentricity for all the cross section to remain in compression (no tension) is defined as the Kern point; the middle-third rule is to design such that the load remains within the diamond formed from the middle third of the width and middle third of the depth (kern area)
- Euler buckling load is $P_{cr} = \pi^2 EI/L^2$; increasing the length reduces the load capacity to 1/4
- Radius of gyration is the square root of I/A and shows up in the critical buckling stress definition = $\pi^2 E/(L/r)^2$ where L/r is the slenderness ratio
- The effective length can be determined from the end conditions providing a factor k which is multiplied by the length; it can be less than one or greater (like a flagpole)
- The critical load must be determined for BOTH axes and the capacity is governed by the SMALLER of the two
- Bracing reduces the effective length; the longer unbraced length will determine the critical load
- Behavior of intermediate column lengths (those between crushing and buckling) are lower than theoretical because it is partly elastic and partly inelastic buckling
- Design criteria:
  - reduce kL/r by increasing r(I)
  - bracing reduces kL as does changing the end conditions
  - efficiency is related to getting similar capacities in each direction, so the ratio of I_x to I_y is often important (or r_x to r_y)
  - increasing bracing can increase cost and construction
- Allowable/limit stress depends on the length and cross section, so design (choosing the size) is not straightforward; can estimate size and evaluate

Updated material

- The critical buckling load is defined as the force that is just big enough to hold the pin-ended column in a slightly deformed – curved – shape; the curved shape is from a moment P(y) related to EI
- The calculus for the differential equations results in the constants of integration – including sin!
- The first mode of buckling results in the critical load: $P_{cr} = \frac{\pi^2 EI}{L^2}$ also known as Euler's buckling equation; deflected shape is a sine curve
- The end (or boundary) conditions of the column will change the constants of integration