Case Study in Steel
adapted from Structural Design Guide, Hoffman, Gouwens, Gustafson & Rice., 2nd ed.

Building description

The building is a one-story steel structure, typical of an office building. The figure shows that it has three 30 ft. bays in the short direction and a large number of bays in the long direction. Some options for the structural system include fully restrained with rigid connections and fixed column bases, simple framing with “pinned” connections and column bases requiring bracing against sideway, and simple framing with continuous beams and shear connections, pinned column bases and bracing against sideway. This last situation is the one we’ll evaluate as shown in Figure 2.5(c).

Loads

Live Loads:
Snow on Roof: 30 lb/ft² (1.44 kPa)

Wind: 20 lb/ft² (0.96 kPa)

Dead Loads:
Roofing: 8 lb/ft² (0.38 kPa)
Estimated decking: 3 lb/ft² (0.14 kPa)
Ceiling: 7 lb/ft² (0.34 kPa)
Total: 18 lb/ft² (0.86 kPa)

Materials

A36 steel for the connection angles (F_y = 36 ksi, F_u = 58 ksi) and A992 (F_y = 50 ksi) for the beams and columns (F_y = 50 ksi) series open web joists and roof deck.

Decking:

Decking selection is typically allowable stress design. Tables will give allowable total uniform load (taking self weight into account) based on stresses and deflection criteria for typical spans and how many spans are supported. The table (and description) for a Vulcraft 1.0 E deck is provided.
Areas in gray are governed by live load roof deflection.

The total load with snow and roofing = 30 psf + 8 psf = 38 psf.

**VERTICAL LOADS FOR TYPE 1.0E**

<table>
<thead>
<tr>
<th>No. of Spans</th>
<th>Deck Type</th>
<th>Allowable Total (Dead + Live) Uniform Load (PSF)</th>
<th>Span (ft)</th>
<th>C. to C. of Support</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>E26</td>
<td>2’-10</td>
<td>178</td>
<td>107</td>
</tr>
<tr>
<td></td>
<td>E24</td>
<td>3’-5</td>
<td>249</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td>E22</td>
<td>3’-10</td>
<td>316</td>
<td>187</td>
</tr>
<tr>
<td></td>
<td>E20</td>
<td>4’-2</td>
<td>379</td>
<td>224</td>
</tr>
<tr>
<td>2</td>
<td>E26</td>
<td>3’-4</td>
<td>273</td>
<td>189</td>
</tr>
<tr>
<td></td>
<td>E24</td>
<td>4’-0</td>
<td>396</td>
<td>275</td>
</tr>
<tr>
<td></td>
<td>E22</td>
<td>4’-6</td>
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<td>357</td>
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<td></td>
<td>E20</td>
<td>5’-0</td>
<td>634</td>
<td>440</td>
</tr>
<tr>
<td>3</td>
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<td>3’-4</td>
<td>310</td>
<td>198</td>
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<tr>
<td></td>
<td>E24</td>
<td>4’-0</td>
<td>469</td>
<td>276</td>
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<tr>
<td></td>
<td>E22</td>
<td>4’-6</td>
<td>586</td>
<td>344</td>
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<td></td>
<td>E20</td>
<td>5’-0</td>
<td>707</td>
<td>413</td>
</tr>
</tbody>
</table>

Notes:
1. Load tables are calculated using sectional properties based on the steel design thickness shown in the Steel Deck Institute (SDI) Design Manual.
2. Loads shown in the shaded areas are governed by the live load deflection not in excess of 1/240 of the span.

Open Web Joists:

Open web joist selection is either based on allowable stress design or LRFD resistance for flexure (not for deflection). The total factored distributed load for joists at 6 ft on center will be:

\[
w_{total} = (1.2 \times 18\text{lb/ft}^2 + 1.6 \times 30 \text{ lb/ft}^2)(6 \text{ ft}) + 1.2(8 \text{ lb/ft estimated}) = 427.2 \text{ lb/ft} \quad \text{(with 1.2}D + 1.6(L, or } L_r, \text{ or } S, \text{ or } R) \text{ by catalogue)}
\]

\[
w_{live} = 30 \text{ lb/ft}^2(6 \text{ ft}) = 180 \text{ lb/ft}
\]
Deflection will limit the selection, and the most lightweight choice is the 22K4 which weighs approximately 8 lb/ft. Special provisions for bridging are required for the shaded area lengths and sections.

**Continuous Beams:**

LRFD design is required for the remaining structural steel for the combinations of load involving Dead, Snow and Wind. The bracing must be designed to resist the lateral wind load.

The load values are:

- For D: \( w_D = 18 \text{ lb/ft}^2 \cdot 30 \text{ ft} + (8 \text{ lb/ft} \cdot 30 \text{ ft})/6 \text{ ft} = 580 \text{ lb/ft} \)
- For S: \( w_S = 30 \text{ lb/ft}^2 \cdot 30 \text{ ft} = 900 \text{ lb/ft} \)
- For W: \( w_W = 20 \text{ lb/ft}^2 \cdot 30 \text{ ft} = 600 \text{ lb/ft} \) (up or down)
  and laterally \( V = 600 \text{ lb/ft}(15\text{ft}/2) = 4500 \text{ lb} \)

These DO NOT consider self weight of the beam.

The applicable combinations for the tributary width of 30 ft. are:

1. \( 1.4D \quad w_u = 1.4(580 \text{ lb/ft}) = 812 \text{ lb/ft} \)
2. \( 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R) \quad w_u = 1.2(580 \text{ lb/ft}) + 0.5(900 \text{ lb/ft}) = 1146 \text{ lb/ft} \)
3. \( 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (L \text{ or } 0.8W) \quad w_u = 1.2(580 \text{ lb/ft}) + 1.6(900 \text{ lb/ft}) + 0.8(600 \text{ lb/ft}) = 2616 \text{ lb/ft} \)
4. \( 1.2D + 1.6W + L + 0.5(L_r \text{ or } S \text{ or } R) \quad w_u = 1.2(580 \text{ lb/ft}) + 1.6(600 \text{ lb/ft}) + 0.5(900 \text{ lb/ft}) = 2106 \text{ lb/ft} \)
5. \( 1.2D + 1.0E + L + 0.25S \quad w_u = 1.2(580 \text{ lb/ft}) + 0.25(900 \text{ lb/ft}) = 921 \text{ lb/ft} \)
6. \( 0.9D + 1.6W + 1.6H \quad w_u = 0.9(580 \text{ lb/ft}) + 1.6(-600 \text{ lb/ft}) \) \([\text{uplift}] = -438 \text{ lb/ft} \) (up)

\( L, R, L_r, E \text{ & } H \) don’t exist for our case.

For the largest load case, the shear & bending moment diagrams are:
For the beams, we know that the maximum unbraced length is 6 ft. For the middle 6 feet of the end span, the moment is nearly uniform, so $C_b = 1$ is acceptable ($C_b = 1.08$ for constant moment). For the interior span, $C_b$ is nearly 1 as well.

Choosing a W18x35 ($M_u = 229$ k-ft) for the end beams, and a W12x30 ($M_u = 158$ k-ft) for the interior beam, the self weight can be included in the total weight. The diagrams change to:

Check beam shear: $V_u \leq \phi V_n = 1.0(0.6F_{yw}A_w)$

Exterior $V_u = 34.67$ k $\leq 1.0(0.6)(50 \text{ ksi})(17.1 \text{ in.})(0.3 \text{ in.}) = 153.9$ k OK

W18x35: $d = 17.7$ in., $t_w = 0.3$ in., $I_x = 510$ in.$^4$

Interior $V_u = 45.05$ k $\leq 1.0(0.6)(50 \text{ ksi})(12.3 \text{ in.})(0.26 \text{ in.}) = 95.94$ k OK

W12x30: $d = 12.3$ in., $t_w = 0.26$ in., $I_x = 238$ in.$^4$
Check deflection (NO LOAD FACTORS) for total and live load (gravity and snow).

**Exterior Beam:** worst deflection is from no live load on the center span:

Maximum $\Delta_{\text{total}} = 2.20$ in.
Is $\Delta_{\text{total}} \leq L/240 = 360 \text{ in.}/240 = 1.5$ in.? NO GOOD
We need an $I \geq (2.20\text{ in.}/1.5\text{ in.})(510 \text{ in.}^4) = 748 \text{ in.}^4$

Maximum $\Delta_{\text{live}} = 1.86$ in.
Is $\Delta_{\text{live}} \leq L/360 = 360 \text{ in.}/360 = 1.0$ in.? NO GOOD
We need an $I \geq (1.86\text{ in.}/1.0\text{ in.})(510 \text{ in.}^4) = 949 \text{ in.}^4$

The W21x48 looks promising, but it has a note that it exceeds the compact limit for flexure.

Choose a W21 x 50 ($I_x = 984 \text{ in.}^4$) (because the W21x48 would require extra work!)

Now, $\Delta_{\text{live}} = 1.07$ in., which is reasonable close.

**Interior Beam:** worst deflection is from load on all spans:

Maximum $\Delta_{\text{total}}$ (at midspan) = 1.31 in.
Is $\Delta_{\text{total}} \leq L/240 = 360 \text{ in.}/240 = 1.5$ in.? OK

Maximum $\Delta_{\text{live}}$ (at midspan) = 0.94 in.
Is $\Delta_{\text{live}} \leq L/360 = 360 \text{ in.}/360 = 1.0$ in.? OK
Columns:

The load in the interior columns: $P_u = 85 \text{ kips (sum of the shears)}$. This column will see minimal eccentricity from the difference in shear and half the column depth as the moment arm.

The load in the exterior columns: $P_u = 35 \text{ kips}$. These columns will see some eccentricity from the beam shear connections. We can determine this by using half the column depth as the eccentricity distance.

The effective length of the columns is 15 ft (no intermediate bracing). Table 4-1 shows design strength in kips for W8 shapes (the smallest). The lightest section at 15 feet has a capacity of 230 k; much greater than what we need even with eccentricity.

The exterior column connection moment (unmagnified) when the W8x31 depth = 8.0 in.

$$\left(35k \times \frac{8.0\text{in}}{2}\times\frac{1\text{ft}}{12\text{in}}\right) = 11.7 \text{k-ft}.$$

The capacity of a W8x31 with an unbraced length of 15 ft (from another beam chart) = 114 \text{k-ft}.

For $\frac{P}{P_c} < 0.2$:

$$\frac{P_u}{2\phi_u P_n} + \left(\frac{M_{nx}}{\phi_b M_{nx}} + \frac{M_{ny}}{\phi_b M_{ny}}\right) \leq 1.0$$

$$\frac{35k}{230k} = 0.15 < 0.2 : \frac{35k}{2(230k)} + \left(\frac{11.7\text{k-ft}}{114\text{k-ft}}\right) = 0.179 \leq 1.0$$

so OK for eccentric loading of the beam-column (but we knew that).

Beam Shear Splice Connection:

For this all-bolted single-plate shear splice, $R_u = 35 \text{ kips}$

W21x50: $d = 20.8 \text{ in.}, t_w = 0.38 \text{ in}$.

W12x30: $d = 12.3 \text{ in.}, t_w = 0.26 \text{ in}$.

The plate material is A36 with $F_y = 36 \text{ ksi}$ and $F_u = 58 \text{ ksi}$. We need to check that we can fit a plate within the fillets and provide enough distance from the last holes to the edge.

For the W12x30, $T = 10.125 \text{ in.}$, which limits the plate height.

For a plate, $s$ (hole spacing) = 3” and minimum edge distance is 1½”.
For ¾ in. diameter A325-N bolts and standard holes without a concern for deformation of the holes, the capacity per bolt is:

shear: \[ R_n \leq \phi R_n \quad \phi = 0.75, \quad R_n = F_n A_n, \quad \text{where} \quad F_n = 54 \text{ ksi} \]

\[
35k \leq n \left( 0.75 \right) \left( 54 \text{ksi} \right) \left( \frac{0.75\text{in.}}{4} \right)
\]

so \( n \geq 1.96 \). Use 2 bolts (1 @ 3 in. + 2 @ 1.25 in. ≈ 5.5 in. < 10.125 in.).

bearing for 2 rows of bolts: depends on thickness of thinnest web (t=0.26 in.) and the connected material

\[ R_u \leq \phi R_n \quad \phi = 0.75, \quad R_n = 1.5 L_c t F_u \leq 3.0 d t F_u \]

\[ L_c = 1.75 \text{ in. from the vertical edge of the beam to the edge of a hole} \]

\[
35k \leq 2 \text{bolts} \left( 0.75\left(1.5\right) \left( 1.75\text{in.} \right) \left( 0.26\text{in} \right) \left( 65\text{ksi} \right) \right) = 38.0 \text{ k} \\
\leq 2 \text{bolts} \left( 0.75\left(3\right) \left( 0.75\text{in} \right) \left( 0.26\text{in} \right) \left( 65\text{ksi} \right) \right) = 57.0 \text{ k} \text{ OK}
\]

If the spacing between the holes across the splice is 4 in., the eccentricity, \( e_x \), is 2 inches. We need to find \( C \), which represents the number of bolts that are effective in resisting the eccentric shear force.

\[
C_{min} = \frac{35k}{0.75(54\text{ksi})\left(0.75\text{in.}\right)^2\pi/4} = 1.95 \quad \text{(which we found as} \quad n)\]

\( C \) off the table is 2.54 bolts which is more than the minimum of 1.95 (which is why we have 2). OK.

If the plate is 3/8 in. thick x 8 in. wide x 9 in. tall, check bolt bearing on plate:

\[ \phi R_n = 2.4 d t F_u \quad \text{(per bolt)} \]

2 bolts \( 2.4 \left(0.75\text{ in.}\right) \left( 0.375 \text{ in.} \right) \left( 58 \text{ ksi} \right) = 78.3 \text{ k} > 35 \text{ k} \quad \text{OK} \]
Check flexure of the plate:

\[ M_n = \frac{R_a e}{2} = \frac{35k \times 4in}{2} = 70.0 \text{ k-in} \]

yielding capacity:

\[ \phi M_n = \phi F_y S_x \phi = 0.9 \text{ (5.5 in. tall section, 3/8 in. thick)} \]

\[ 0.9(36ksi) \left( \frac{0.375\text{in}(5.5in)^2}{6} \right) = 61.25 \text{ k-in} > 70.0 \text{ k-in} \text{ NOT OK} \]

with 6 in. tall, \( \phi M_n = 72.9 \text{ k-in} \)

rupture

\[ \phi M_n = \phi F_y S_{net} \phi = 0.75 \]

\[ S_{net} = \frac{I_{net}}{c} \text{ and can be looked up or calculated} = 1.74 \text{ in}^3 \]

\[ 0.75(58ksi)(1.74\text{in}^3) = 75.7 \text{ k-in} > 70.0 \text{ k-in} \text{ OK} \]

Check shear yielding of the plate: \( R_u \leq \phi R_n \phi = 1.00 \quad R_n = 0.6F_yA_g \)

\[ (1.00)[0.6(36ksi)(6\text{ in.})(0.375 \text{ in.})] = 48.6 \text{ k} > 35 \text{ k} \text{ OK} \]

Check shear rupture of the plate: \( R_u \leq \phi R_n \phi = 0.75 \quad R_n = 0.6F_yA_{nv} \)

for ¾” diameter bolts, the effective hole width is \((0.75 + 1/8) = 0.875 \text{ in.} : \)

\[ (0.75)[0.6(58ksi)(6\text{ in.} - 2 \times 0.875 \text{ in.})(0.375 \text{ in.})] = 41.6 \text{ k} > 35 \text{ k} \text{ OK} \]

Check block shear rupture of the plate: \( R_u \leq \phi R_n \phi = 0.75 \)

\[ R_n = 0.6F_yA_{nv} + U_{bs}F_uA_{nt} \leq 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \]

with \( U_{bs} = 0.5 \) when the tensile stress is non-uniform. (The tensile stress switches direction across the splice.) (and assuming 2 in. of width to the center of the bolt hole)

\[ R_n = 0.60(58ksi)(0.375\text{in})[1.5\text{in} + 3\text{in} - 1.5^{holes}(0.875)] + \]

\[ 0.5(58ksi)(0.375\text{in})(2\text{in} - 0.875\text{in}/2) = 58.6.9k \]

\[ \leq 0.6(36ksi)(0.375\text{in})(1.5\text{in} + 3\text{in}) + 0.5(36ksi)(0.375\text{in})(2\text{in} - 0.875\text{in}/2) = 47.0k \]

\[ 35 \text{ k} < 0.75(47.0 \text{ k}) = 35.2 \text{ k} \text{ OK} \]

Column Base Plate:

Column base plates are designed for bearing on the concrete (concrete capacity) and plastic hinge development from flexure because the column “punches” down the plate and it could bend upward near the edges of the column (shown as \( 0.8b_f \) and \( 0.95d \)). The plate dimensions are \( B \) and \( N \). The concrete has a compressive strength, \( f'_c = 3 \text{ ksi} \).
For W8 x 31: d = 8.0 in., b_{f} = 8.0 in., and if we provide width to put in bolt holes, we could use a 12 in. by 12 in. plate (allowing about 2 inches each side). We will look at the interior column load of 85 k.

**minimum thickness:** \( t_{\text{min}} = l \sqrt{\frac{2P_{u}}{0.9F_{y}BN}} \)

where \( l \) is the larger of \( m \), \( n \) and \( \lambda n' \)

\[
m = \frac{(N - 0.95d)}{2} = \frac{(12 \text{ in.} - 0.95 \times 8.0 \text{ in.})}{2} = 2.2 \text{ in.}
\]

\[
n = \frac{(B - 0.8b_{f})}{2} = \frac{(12 \text{ in.} - 0.8 \times 8.0 \text{ in.})}{2} = 2.8 \text{ in.}
\]

\[
n' = \frac{\sqrt{db_{f}}}{4} = \frac{\sqrt{8.0 \text{ in} \times 8.0 \text{ in.}}}{4} = 2.0 \text{ in.}
\]

\( \lambda \) is derived from a term \( X \) which takes the bounding area of the column, the perimeter, the axial force, and the concrete compressive strength into account:

\[
X = \frac{4db_{f}}{(d + b_{f})^2}, \quad P_{u} = \frac{4db_{f}}{(d + b_{f})^2}, \quad P_{c} = \frac{P_{u}}{\phi_{c}(0.85f'_{c})BN} = \frac{4 \times 8.0 \text{ in} \times 8.0 \text{ in.}}{(8.0 \text{ in} + 8.0 \text{ in.})^2}, \quad 85k \\
= \frac{85k}{0.6(0.85 \times 3 \text{ksi})12 \text{ in} \times 12 \text{ in.}}
\]

\[
\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1 - X}} \leq 1 = \frac{2\sqrt{0.386}}{1 + \sqrt{1 - 0.386}} = 0.697 \quad \text{so} \quad \lambda n' = (0.697)(2.0 \text{ in.}) = 1.39 \text{ in.}
\]

\[
t_{p} = l \sqrt{\frac{2P_{u}}{0.9F_{y}BN}} = (2.8 \text{ in.}) \sqrt{\frac{2 \times 85k}{0.9(36 \text{ ksi})(12 \text{ in.})(12 \text{ in.})}} = 0.513 \text{ in.}
\]

Use a 9/16 in. thick plate.

The anchor bolts must also be able to resist lateral shear. There also is friction between the steel and concrete to help. The International Building Code provided specifications for minimum edge distances and anchorage.

**Continuous Beam Over Interior Column:**

The design for this connection will involve a bearing plate at the top of the column, with a minimum number of bolts through the beam flanges to the plate. Because there will be high local compression, stiffener plates for the web will need to be added (refer to a plate girder design). Flexure with a reduced cross section area of the flanges should be checked.