Concrete Beam Design

- composite of concrete and steel
- American Concrete Institute (ACI)
  - design for maximum stresses
  - limit state design
  - service loads x load factors
  - concrete holds no tension
  - failure criteria is yield of reinforcement
  - failure capacity x reduction factor
  - factored loads < reduced capacity
- concrete strength = $f'_c$
Concrete Construction

- cast-in-place
- tilt-up
- prestressing
- post-tensioning
Concrete Beams

- types
  - reinforced
  - precast
  - prestressed

- shapes
  - rectangular, I
  - T, double T’s, bulb T’s
  - box
  - spandrel
Concrete Beams

- **shear**
  - vertical
  - horizontal
  - combination:
    - tensile stresses at 45°

- **bearing**
  - crushing
Concrete

- low strength to weight ratio
- relatively inexpensive
  - Portland cement
    - types I - V
  - aggregate
    - course & fine
  - water
  - admixtures
    - air entraining
    - superplasticizers
Concrete

• hydration
  – chemical reaction
  – workability
  – water to cement ratio
  – mix design

• fire resistant

• cover for steel

• creep & shrinkage
Concrete

- placement (not pouring!)
- vibrating
- screeding
- floating
- troweling
- curing
- finishing
Reinforcement

- **Deformed steel bars (rebar)**
  - Grade 40, $F_y = 40$ ksi
  - Grade 60, $F_y = 60$ ksi - most common
  - Grade 75, $F_y = 75$ ksi
  - US customary in # of 1/8” $\phi$

- **Longitudinally placed**
  - bottom
  - top for compression reinforcement
Reinforcement

- prestressing strand
- post-tensioning
- stirrups
- detailing
  - development length
  - anchorage
  - splices
Composite Beams

- concrete
  - in compression
- steel
  - in tension
- shear studs
Behavior of Composite Members

- plane sections remain plane
- stress distribution changes

\[ f_1 = E_1 \varepsilon = -\frac{E_1 y}{R} \]
\[ f_2 = E_2 \varepsilon = -\frac{E_2 y}{R} \]
Transformation of Material

• \( n \) is the ratio of \( E \)'s

\[
n = \frac{E_2}{E_1}
\]

• effectively widens a material to get same stress distribution
Stresses in Composite Section

- with a section transformed to one material, new $I$
  - stresses in that material are determined as usual
  - stresses in the other material need to be adjusted by $n$

$$n = \frac{E_2}{E_1} = \frac{E_{\text{steel}}}{E_{\text{concrete}}}$$

$$f_c = -\frac{M_y}{I_{\text{transformed}}}$$

$$f_s = -\frac{M_{yn}}{I_{\text{transformed}}}$$
Reinforced Concrete - stress/strain

Stresses in the concrete above the neutral axis are compressive and nonlinearly distributed. In the tension zone below the neutral axis, the concrete is assumed to be cracked and the tensile force present to be taken up by reinforcing steel.

Working stress analysis. (Concrete stress distribution is assumed to be linear. Service loads are used in calculations.)

Actual stress distribution near ultimate strength (nonlinear).

Ultimate strength analysis. (A rectangular stress block is used to idealize the actual stress distribution. Calculations are based on ultimate loads and failure stresses.)

FIGURE 6-37 Reinforced concrete beams.
Reinforced Concrete Analysis

• for stress calculations
  – steel is transformed to concrete
  – concrete is in compression above n.a. and represented by an equivalent stress block
  – concrete takes no tension
  – steel takes tension
  – force ductile failure
Location of n.a.

- ignore concrete below n.a.
- transform steel
- same area moments, solve for $x$

\[
bx \cdot \frac{x}{2} - nA_s (d - x) = 0
\]
**T sections**

- n.a. equation is different if n.a. below flange

\[
b_f h_f \left(x - \frac{h_f}{2}\right) + (x - h_f)b_w \frac{(x - h_f)}{2} - nA_s (d - x) = 0
\]
ACI Load Combinations*

- 1.4D
- 1.2D + 1.6L + 0.5(L_r or S or R)
- 1.2D + 1.6(L_r or S or R) + (1.0L or 0.5W)
- 1.2D + 1.0W + 1.0L + 0.5(L_r or S or R)
- 1.2D + 1.0E + 1.0L + 0.2S
- 0.9D + 1.0W
- 0.9D + 1.0E

*can also use old ACI factors
Reinforced Concrete Design

- stress distribution in bending

![Diagram showing stress distribution in bending](image)

Actual stress: $\frac{0.85f'_c}{\beta_1 c}$

Whitney stress block: $\frac{a/2}{C}$

Wang & Salmon, Chapter 3
**Force Equations**

- \( C = 0.85 f'_c ba \)
- \( T = A_s f_y \)

where

- \( f'_c = \text{concrete compressive strength} \)
- \( a = \text{height of stress block} \)
- \( \beta_1 = \text{factor based on } f'_c \)
- \( c = \text{location to the n.a.} \)
- \( b = \text{width of stress block} \)
- \( f_y = \text{steel yield strength} \)
- \( A_s = \text{area of steel reinforcement} \)

\[
\beta_1 = 0.85 - \left( \frac{f'_c - 4000}{1000} \right)(0.05) \geq 0.65
\]
Equilibrium

- \( T = C \)
- \( M_n = T(d-a/2) \)
  - \( d = \) depth to the steel n.a.
- with \( A_s \)
  - \( a = \frac{A_s f_y}{0.85 f'_c b} \)
  - \( M_u \leq \phi M_n \quad \phi = 0.9 \) for flexure*
  - \( \phi M_n = \phi T(d-a/2) = \phi A_s f_y (d-a/2) \)

\* \( \phi = 0.65 + (\varepsilon_i - \varepsilon_y) \frac{0.25}{(0.005 - \varepsilon_y)} \geq 0.65 \)
Over and Under-reinforcement

- **over-reinforced**
  - steel won’t yield

- **under-reinforced**
  - steel will yield

- **reinforcement ratio**

  \[
  \rho = \frac{A_s}{bd} 
  \]

  - use as a design estimate to find \( A_s, b, d \)
  
  - max \( \rho \) is found with \( \varepsilon_{\text{steel}} \geq 0.004 \) (not \( \rho_{\text{bal}} \))
  
  - *with \( \varepsilon_{\text{steel}} \geq 0.005, \phi = 0.9 \)

http://people.bath.ac.uk/abstji/concrete_video/virtual_lab.htm
As for a Given Section

- several methods
  - guess a and iterate
    1. guess a (less than n.a.)
    2. \[ A_s = \frac{0.85 f'_b a}{f_y} \]
    3. solve for a from \( M_u = \phi A_s f_y (d-a/2) \)
    \[
    a = 2 \left( d - \frac{M_u}{\phi A_s f_y} \right) \]
    4. repeat from 2. until a from 3. matches a in 2.
**As for a Given Section (cont)**

- chart method
  - Wang & Salmon Fig. 3.8.1 \( R_n \) vs. \( \rho \)

1. calculate \( R_n = \frac{M}{bd^2} \)

2. find curve for \( f'_c \) and \( f_y \) to get \( \rho \)

3. calculate \( A_s \) and \( a \)

- simplify by setting \( h = 1.1d \)
Reinforcement

- min for crack control
- required
  \[ A_s = \frac{3\sqrt{f_c'}}{f_y} (bd) \]
- not less than
  \[ A_s = \frac{200}{f_y} (bd) \]
- \( A_{s\text{-max}} \): \( a = \beta_1 (0.375d) \)
- typical cover
  - 1.5 in, 3 in with soil
- bar spacing
Shells
Annunciation Greek Orthodox Church

• Wright, 1956
Annunciation Greek Orthodox Church

• Wright, 1956
Cylindrical Shells

- can resist tension
- shape adds “depth”

• not vaults
• barrel shells
Kimball Museum, Kahn 1972
Kimball Museum, Kahn 1972

• outer shell edges
Kimball Museum, Kahn 1972

- skylights at peak
Approximate Depths

<table>
<thead>
<tr>
<th>Slabs (poured in place)</th>
<th>Simply supported</th>
<th>One end continuous</th>
<th>Both ends continuous</th>
<th>Cantilever L/12</th>
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<td>Beams (poured in place)</td>
<td>Simply supported</td>
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<td>Both ends continuous</td>
<td>Cantilever L/10</td>
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<td>Pan joist system</td>
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<td>Folded plate</td>
<td>L/8 – L/15</td>
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<td>Barrel shell</td>
<td>L/8 – L/15</td>
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<td>Planks (precast)</td>
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<td>Channels (precast)</td>
<td>L/20 – L/28</td>
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<tr>
<td>Tees (precast)</td>
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<tr>
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<td>L/30 – L/40</td>
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<td>Flat slab</td>
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<td>Two-way beam and slab</td>
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<td>Waffle slab</td>
<td>L/23 – L/35</td>
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<tr>
<td>Dome (poured in place)</td>
<td>L/4 – L/8</td>
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